Team Round

LMT Spring 2026

May 9, 2026

1. **[20]** Sam and Eddie love eating cakes. Sam eats 80% of his cake, while Eddie eats 52% of his cake. Interestingly, they ate the same amount of cake. If the size of Sam's cake was x% of the size of Eddie's cake before either of them ate their cakes, find x.

2. **[20]** Find the volume of the region that is swept out by a unit cube when it undergoes a 360° rotation about one of its edges.

3. **[25]** Six perfectly logical people: Peter, William, Isabella, Jonathan, Jerry, and Ben, are playing a variation of Castlefall. Specifically, three distinct integers from 1 to 8 are chosen, and each integer is given to exactly two people, forming three teams of two. Each person only sees their own number, and everyone is aware of how the numbers were distributed. They have the following dialogue:

   - Peter: "My number is a multiple of 4."
   - William: "My number is prime."
   - Isabella: "My number is a factor of 5."
   - Jonathan: "My number is even."
   - Jerry: "My number is less than 5."
   - Ben: "My number is odd."
   - Peter: "I know who my partner is."
   - William: "I can identify the number each person has."

   Find the numbers that are given to each of Peter, William, Isabella, Jonathan, Jerry, and Ben as an ordered list of six integers in that order.

4. **[30]** A number n satisfies the property that n is 36 times the sum of its digits. Find the sum of all possible values of n.

5. **[30]** Let a, b, c be nonzero real numbers such that

   $$a = b^{2026} + c^{2026}$$
   $$b = a^{2026} + c^{2026}$$
   $$c = a^{2026} + b^{2026}.$$

   Find the product of all possible values of a.

6. **[35]** Find the number of ways to replace each _ with either $+$, $-$, or $\times$ in the expression

   $$1\ \_\ 2\ \_\ 3\ \_\ 4\ \_\ 5\ \_\ 6\ \_\ 7$$

   such that the result evaluates to an odd number.

7. **[40]** Let ABC be a triangle with $AB = 6$. Let M be the midpoint of BC. A circle with its center on AC is tangent to AB at A and BC at M. The circle intersects AC at distinct points A and X. Find CX.

8. **[40]** Let p, q, r be three primes, and define

   $$k = \frac{pqr}{(p-2)(q-2)(r-2)}.$$

   Find the sum of all possible integer values of k.

9. **[40]** There are 2026 people at LMT, all of which go to Clarke, Diamond, or LHS.

   - Before the individual rounds, every pair of people from the same school shake hands.
   - Before the team round, every Clarke student shakes hands with every non-Clarke student.
   - Before the guts round, every Clarke student shakes hands with every Diamond student.

   Given that there were $2050312 = 2026 \times 1012$ hand shakes total, find the number of Clarke students at LMT.

10. **[45]** Define the function
$$f(t) = 20\left\lfloor \frac{t}{20} \right\rfloor - 26\left\lfloor \frac{t}{26} \right\rfloor.$$
Suppose that $f(t)$ has a minimum of m and a maximum of M. Find (m, M).

11. **[50]** Let a_n, b_n be sequences of numbers satisfying the recurrences
$$\frac{a_{n+1}}{a_n} = a_n^2 + 3b_n^2 \qquad \text{and} \qquad \frac{b_{n+1}}{b_n} = b_n^2 + 3a_n^2.$$
Given that $a_0 = 2$ and $b_0 = 1$, find a_{2026}.

12. **[50]** Sam writes some of the numbers from 1 to 10 onto a chalkboard (at most one of each number and he writes at least one number). In one move, he chooses 2 distinct elements a and b currently on the chalkboard, erases them both, and writes down
$$\frac{\operatorname{lcm}(a, b)}{\gcd(a, b)}.$$
The process is repeated until one number remains on the board. Find the number of initial states such that there is a process in which the final number is a perfect square.

13. **[55]** Let ABC be a triangle, let M be the midpoint of BC, and let G be the centroid of ABC. Suppose that the circumcircle of $\triangle BGM$ intersects AB at P and the circumcircle of $\triangle CGM$ intersects AC at Q. Given that $AP = \frac{1}{8}$, $AG = \frac{2}{11}$, and $AQ = \frac{1}{9}$, find the perimeter of quadrilateral $APGQ$.

14. **[60]** Suppose that integers a, b, and c satisfy
$$10\left(a^2 + b^2 + c^2\right) + 6(ab + bc + ca) = 2^6 \cdot 5 \cdot 23.$$
Given that $0 \le a \le b \le c$, there exists a unique solution (a, b, c) to the equation. Find the value of $a + b + c$.

15. **[60]** In triangle ABC, D lie on the circumcircle and E lies on BC such that $AD \perp BC$ and $\angle BAE = \angle EAC$. Suppose $DE \perp AC$. Given that $AB = 6$ and $AC = 7$, find BC.