Speed Round

LMT Spring 2026

May 9, 2026

1. **[6]** Let $a \star b = \frac{a+b}{2}$. Find the value of
$$((1 \star 2) \star (3 \star 4)) \star ((5 \star 6) \star (7 \star 8)).$$

2. **[6]** Let $MATH$ be a parallelogram with area 2026. Let L be the midpoint of MA. Find the area of LMT.

3. **[6]** Find the maximum possible value of $a^b + c - d$ where the ordered quadruple (a, b, c, d) is a permutation of $(2, 0, 2, 6)$.

4. **[6]** Three standard fair 6-sided dice are rolled. Find the probability that the product of numbers on the top faces is prime.

5. **[6]** Cup A contains a $1:5$ mixture of sugar to water and cup B contains a $4:7$ mixture of sugar to water. Cup C is a mixture of the solutions from Cup A and Cup B in a $2:3$ ratio. Find the number of grams of sugar in 660 grams of Cup C solution.

6. **[6]** Let ABC be an equilateral triangle with side length 1. Let D be the foot of the altitude from A to BC. The circle with diameter AD intersects AB and AC at points X and Y, respectively. Find XY.

7. **[6]** Find the number of ways to arrange the numbers 1, 2, 3, 4, and 5 in a row such that the sum of any two adjacent numbers is not divisible by 3.

8. **[6]** Let a, b, c be relatively prime positive integers satisfying
$$a + b = c \qquad \text{and} \qquad 20a = 2b + 6c.$$
Find the value of $a + b + c$.

9. **[6]** Suppose primes p and q satisfy the property that $(p+1)(q+1)$ is not divisible by any perfect square greater than 2. Find the smallest possible value of $p + q$.

10. **[6]** Rectangle $ABCD$ and triangle ABE share side AB and do not overlap. If $AB = 5, AE = 4, BE = 3$, and the area of $\triangle CDE$ is 21, find BC.

11. **[6]** Find the remainder when 10^{2026} is divided by 9801.

12. **[6]** Math team has 4 freshman, 4 sophomores, and 4 juniors. Find the number of ways there are to split them into 3 groups of 4 such that each group has at least one person from each grade.

13. **[6]** Let f be a polynomial of degree 3 with leading coefficient 1. Suppose $f(2) = 67, f(-2) = 41$. Find $f(10) - f(-10)$.

14. **[6]** The numbers 1 through 9 are placed randomly in an 3×3 grid. Find the expected number of 2×2 subgrids which contain 4 consecutive numbers.

15. **[6]** The parabola $y = x^2$ is drawn. Two points A, B with integer coordinates are selected along the parabola on opposite sides of $x = 0$ such that the line passing through A and B has y-intercept 7. Find the sum of all possible y-coordinates of A.

16. **[6]** Find the area of a triangle with side lengths of $\sqrt{10001}$, $\sqrt{2501}$, and $\sqrt{2504}$.

17. **[6]** Find the value of
$$\sum_{n=1}^{\infty} \frac{1}{(n+2)n!}.$$

18. **[6]** Find the minimum possible value of
$$4\left\lfloor \frac{b+c}{a} \right\rfloor + 3\left\lfloor \frac{c+a}{b} \right\rfloor + 4\left\lfloor \frac{a+b}{c} \right\rfloor,$$
where a, b, and c are the side lengths of a non-degenerate triangle.

19. **[6]** Find the number of ordered tuples $(a_1, a_2, \dots, a_{10})$ that satisfy the following conditions:
    - $a_i \in \{1,2,3\} \quad (i = 1,2,\dots,10)$.
    - If $i = 1,3,5,7,9$, then $a_i < a_{i+1}$.
    - If $i = 2,4,6,8$, then $a_i \geq a_{i+1}$.

20. **[6]** Let α be a real number picked uniformly at random from $(0,1)$. Find the expected value of
$$\sum_{k=1}^{10} \lfloor k\alpha \rfloor = \lfloor \alpha \rfloor + \lfloor 2\alpha \rfloor + \cdots + \lfloor 10\alpha \rfloor.$$

21. **[6]** A sphere with center O and radius 169 contains points A, B, and C on its surface such that $AB = 104$, $AC = 112$, and $BC = 120$. Let P_1, P_2, and P_3 be the planes tangent to the sphere at A, B, and C respectively. These three planes intersect at a unique point X. Find the value of OX.

22. **[6]** For each color in the set {red, orange, yellow, green, blue, purple}, there are three balls of that color. Find the number of ways to partition the balls into groups of three such that no group contains balls of three different colors.

23. **[6]** Distinct primes p, q, r satisfy
$$2p^{q-1} + 2q^{p-1} = 3pr + 7qr + 2.$$
Find the value of r.

24. **[6]** Let $ABCD$ be a convex quadrilateral with $AB = 9, BC = 15$, and $CD = 13$. Let the diagonals intersect at E. Suppose (AED) and (BCE) intersect AB at points $P \neq A$ and $Q \neq B$. Given $DP \parallel CQ$, find DA.

25. **[6]** Find the value of
$$\sum_{n=0}^{\infty} \frac{2^n}{2026^{2^n} + 1}.$$