

Team Round

LMT Spring 2025

May 3, 2025

1. [20] Find the sum of all integers x for which $|x - 5| + |x - 10|$ is minimized.
2. [25] Suppose x is a positive integer satisfying $100^2 + 2^2 = 20^2 + x^2$. Find x .
3. [25] Jordan sums the numbers from 1 to 100 inclusive. However, he accidentally excludes two numbers from the sum and gets a multiple of 97. Find the maximum possible sum of these two numbers.
4. [30] Find the number of permutations a_1, a_2, a_3, a_4, a_5 of the numbers 2, 3, 4, 5, 6 such that $\frac{a_k}{k}$ is an integer for all $1 \leq k \leq 5$.
5. [30] Primes p, q, r, s satisfy the following equations

$$\begin{aligned}3pq - r &= 51s, \\5qr - p &= 50s, \text{ and} \\7rp - q &= 49s.\end{aligned}$$

Find (p, q, r, s) .

6. [35] William has a bag of red and blue marbles. If he draws a red marble, the probability of drawing a blue marble increases by $\frac{1}{2025}$. Find the least possible number of blue marbles in the bag.
7. [35] Let $ABCD$ be a rectangle, and P a point on the circumcircle of $ABCD$. Suppose $\frac{PA}{PD} = \frac{3}{5}$ and $\frac{PB}{PC} = \frac{7}{8}$. Find $\frac{PA}{PC}$.
8. [40] A four digit base 10 number $\underline{a} \underline{b} \underline{c} \underline{d}$ (potentially with leading zeroes) is a multiple of 99. Suppose that one of the numbers $\underline{a} \underline{b}$ and $\underline{c} \underline{d}$ divides the other. Find the sum of all possible values of $\underline{a} \underline{b} \underline{c} \underline{d}$.
9. [45] Neel travels through all vertices of a regular 21-gon by traveling along diagonals (not sides) and finishes at the vertex where he started. Find the difference between the maximum and minimum number of times his path can intersect itself in the interior of the 21-gon.
10. [45] Let $ABCD$ be a trapezoid with perpendicular diagonals and $AB \parallel CD$. Let E and F be the feet of the perpendiculars from B to CD and E to BD , respectively. Given that F is the centroid of triangle ADE and $AD = 10$, find the area of the trapezoid.
11. [50] Real numbers x and y satisfy the following equations:

$$\begin{aligned}\sqrt{x^2 + y^2} + \sqrt{(x - 3)^2 + (y - 4)^2} &= 5, \\2x^2 + 3xy &= 24.\end{aligned}$$

Find $(x + y)^2$.

12. [50] Mickey Mouse is walking on the Cartesian Plane. He is at the point $(5, 0)$ and is trying to get to his biology classroom at $(0, 0)$. In each part of his journey, he chooses a lattice point and walks to the lattice point in a straight line. Once he arrives at that lattice point, he chooses another lattice point and repeats the process. Given that Mickey Mouse must be walking closer to his biology classroom at all times, find the maximum distance he travels on his journey.
13. [55] Mira partitions the cells of a 5×5 grid into disjoint rectangles. Any vertex on the border (excluding corners) of the 5×5 grid is shared by exactly two rectangles, and any vertex of the 5×5 grid in the interior is shared by exactly 3 rectangles. Find the number of partitions Mira can make.

14. [55] Let $ABCD$ be a quadrilateral with $\angle B = \angle D = 90^\circ$, and $\angle BCA = 30^\circ, \angle DCA = 45^\circ$. Let M be the midpoint of AC and ω be the circumcircle of BMD . The extensions of lines AB, BC, CD, DA intersect ω again at points W, X, Y, Z , respectively. Find $\frac{AB+AD}{WY+XZ}$.
15. [60] In math team, every pair of members are either bros or opps. If the previous person is a bro, then a math team member will make a statement of the same truth value, and if the previous person is an opp, then a math team member will make a statement of the opposite truth value. The five captains of math team have the following conversation:
- William: I'm Muztaba's opp.
 - Ella: Among me, Muztaba, and Jacob there are an even number of pairs of opps.
 - Jacob: Peter and William are opps.
 - Muztaba: Jacob has no opps.
 - Ella: Jacob has three opps.
 - Peter: I'm Jacob's opp.
 - William: Peter has no opps.
 - Peter: Jacob and Ella are not opps.
 - Muztaba: I have more opps than Jacob.

Let W, E, J, M , and P be the number of opps of William, Ella, Jacob, Muztaba, and Peter, respectively. Find $10000W + 1000E + 100J + 10M + P$.