Theme Round Solutions

LMT Fall 2025

December 13th, 2025

Clash Royale

Clash Royale is a mobile game where two players manage their resources to topple the other player's structures. Although it was released in 2016, it became incredibly popular this year. It is also known for the plethora of memes that it inspired, such as "sneaky golem," overleveled mega knight users, and, of course, "HOG RIDERRRRRR."

1. **[6]** Let $CLASH$ be a regular pentagon and $ROYALE$ be a regular hexagon. Find the sum of both possible values of $\angle CLE$ in degrees.

   Proposed by: Samuel Tsui

   Solution. $\boxed{144}$

   The answer is $360-108-120+120-108 = 360-216 = 144$. □

2. **[8]** Archer and Giant, two perfectly logical people, know each other's favorite numbers are positive two-digit integers, and they don't know which.

   Archer: My number has 3 factors including 1 and itself.

   Giant: Ok, my number is less than your number then.

   Archer: I see. My number is definitely more than twice your number, then.

   Giant: Also, my number is divisible by 9.

   Find the positive difference between Archer and Giant's favorite numbers.

   Proposed by: Isabella

   Solution. $\boxed{31}$

   Archer's first sentence means the number is a prime square, so 25 or 49. Giant's first sentence means the number is between 10 and 24. Archer's second sentence means her number is 49. Giant's second sentence reveals her number is 18. □

3. **[10]** At a Clash Royale store, you can buy anywhere from 0 to 25 boxes of mega knights. In the normal store, boxes contain 3 or 12 mega knights; in the evil store, you can boxes contain 2 or 8 mega knights. Find the number of values n such that you can buy n mega knights in the evil store but not in the normal store.

   Proposed by: Derek Yu and Isabella Li

   Solution. $\boxed{65}$

   At the evil store one can reach every even number by adding $2\cdot 1, 2\cdot 2, 2\cdot 3$ to a multiple of 8, which is every number $\le 8\cdot 25 = 200$ except for $8\cdot 23+2\cdot 3 = 190$ $8\cdot 24+2\cdot 2 = 196$ $8\cdot 24+2\cdot 3 = 198$. In the normal store, we can buy any multiple of 3 that we care about (which is under 200). So, we want even numbers ≤ 200 that are not multiples of 6. Using complementary counting, the multiples o 6 are 33, so the complement is 67, and minus 190 and 196 we have 65. □

4. **[12]** Suppose B, A, T, S are the distinct roots of the polynomial $x^4 - x^2 - 1$. Find the value of $B^8 + A^8 + T^8 + S^8$.

   Proposed by: Rohan Danda

Solution. $\boxed{14}$

Observe that $B^4 = B^2 + 1$, so $B^8 = B^4 + 2B^2 + 1 = B^2 + 2$, so we are looking for

$$3(B^2 + A^2 + T^2 + S^2) + 8.$$

However, $B^2 + A^2 + T^2 + S^2 = (B + A + T + S)^2 - 2 \cdot \sum_{\text{cyc}} BA = 2$, so the answer is 14. □

5. **[14]** Let $RAGE$ by a cyclic quadrilateral with perpendicular diagonals such that RG and AE intersect at O. Let L lie on ER such that L, O, and the midpoint of AG are collinear. If $LR = OA = 4$ and $EL = 9$, find the area of LOG.

   Proposed by: Samuel Tsui

Solution. $\boxed{\dfrac{36\sqrt{13}}{13}}$

Suppose $M = (A+G)/2$. Observe that $\angle LOR = \angle MOG = \angle OGM = \angle LEO$ so $OL \perp ER$. Thus, $LO = \sqrt{RL \cdot LE} = 6$. Thus, $OR = 2\sqrt{13}, OE = 3\sqrt{13}$. By Power of a point, $OG = \frac{4 \cdot 3\sqrt{13}}{2\sqrt{13}} = 6$. Thus, $[LOG] = \frac{1}{2} \cdot LO \cdot OG \cdot \sin\angle LOG = 6 \cdot 6 \cdot \frac{1}{\sqrt{13}}$. □

Lebron

LeBron (LeBron James) is an American professional basketball player for the Los Angeles Lakers, the NBA's all-time leading scorer with four championships and four MVP awards, widely regarded as "the goat."

1. **[6]** In the 2025-2026 NBA Season, Lebron has played 6 games and is averaging 14 points per game. Find the number of points he must score in his next game such that he averages 23 points per game over all 7 games.

   Proposed by: Jerry Xu

Solution. $\boxed{77}$

$$7 \cdot 23 - 6 \cdot 14 = 161 - 84 = \boxed{77}.$$

□

2. **[8]** Lebron is in a circular region centered at the origin with radius 3 meters, with defenders at the points $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$. If Lebron is within 1 meter of exactly 2 defenders, he is heavily contested, and if he is within 1 meter of exactly 0 defenders, he is wide open. Given that he is placed uniformly at random inside the circular region, find the probability he is wide open minus the probability he is heavily contested.

   Proposed by: Henry Eriksson

Solution. $\boxed{\dfrac{5}{9}}$

Let A_f be the uncovered area, A_d be the double covered area, and A_w be the area covered by at least one guard. We want to find $\frac{A_f}{9\pi} - \frac{A_d}{9\pi} = \frac{A_f - A_d}{9\pi} = \frac{(A_f + A_w) - (A_d + A_w)}{9\pi}$. Clearly $A_f + A_w = 9\pi$. $A_d + A_w$ counts the area covered by the guards, counting twice the double-covered parts. There is only area 0 covered more than twice, so $A_d + A_w$ is the sum of the areas seen by each guard, or $4 \cdot \pi = 4\pi$. Therefore, our value is $\frac{5\pi}{9\pi} = \boxed{\dfrac{5}{9}}$. □

3. **[10]** Let b, r, o, n be primes satisfying:

$$b \cdot r \cdot o + n = 784$$
$$b + r \cdot o \cdot n = 91.$$

   Find $b + r + o + n$.

   Proposed by: Samuel Tsui

Solution. $\boxed{44}$

Subtracting the second equation from the first gives $(ro-1)(b-n) = 683$. Then, both $ro-1$ and $b-n$ are odd. This means either $r = 2$ or $o = 2$, and also $n = 2$. Suppose $o = 2$. Then, the equations become $2br+2 = 784$ and $b+4r = 101$. The first becomes $br = 391 = 17\cdot 23$. So, if we let $b = 23$ and $r = 17$, then $b+4r = 91$. So our answer is $23+17+2+2 = \boxed{44}$.

□

4. **[12]** Lebron's fans hold up L every 2 seconds, E every 3 seconds, B every 4 seconds, R every 6 seconds, O every 9 seconds, and N every 23 seconds. One second before noon, they start by holding up LEBRON at the same time. Over the next hour (including 1:00:00), for how many seconds are they holding up exactly 5 letters in LEBRON?

   Proposed by: Samuel Tsui, Edwin Zhao, and Evin Liang

   Solution. $\boxed{110}$

   If it is n seconds after noon, then Lebron's fans hold up L if n is a multiple of 2, E if n is a multiple of 3, etc. So this is asking for the amount of integers from 0 to 3600 that are divisible by exactly 5 of 2, 3, 4, 6, 9, and 23. Also note that 0 does not work, so this is the same as the number of integers from 1 to 3600 that are divisible by exactly 4 of 2, 3, 4, 6, 9, and 23.

   This is the same as being divisible by all but one of 2, 3, 4, 6, 9, and 23. The unique one that n is not divisible by cannot be a factor of another of 2, 3, 4, 6, 9, or 23, because then there would be two numbers n is not a multiple of. So the excluded number must be 4, 9, or 23.

   If the excluded number is 23, this is the same as requiring n to be a multiple of $\text{lcm}(2,3,4,6,9) = 36$ but not $36\cdot 23$. So the number is $\lfloor \frac{3600}{36} \rfloor - \lfloor \frac{3600}{36\cdot 23} \rfloor = 100-4 = 96$.

   If the excluded number is 9, this is the same as requiring n to be a multiple of $\text{lcm}(2,3,4,6,23) = 12\cdot 23$ but not $36\cdot 23$. So the number is $\lfloor \frac{3600}{12\cdot 23} \rfloor - \lfloor \frac{3600}{36\cdot 23} \rfloor = 13-4 = 9$.

   If the excluded number is 4, this is the same as requiring n to be a multiple of $\text{lcm}(2,3,6,9,23) = 18\cdot 23$ but not $36\cdot 23$. So the number is $\lfloor \frac{3600}{18\cdot 23} \rfloor - \lfloor \frac{3600}{36\cdot 23} \rfloor = 9-4 = 5$.

   Adding up everything, the total number is $96+9+5 = \boxed{110}$. □

5. **[14]** Every point Lebron scores comes from a 2 point or 3 point shot. Find the number of ways he can score 23 points if he never scores two 3 points shots in a row.

   Proposed by: Samuel Tsui

   Solution. $\boxed{68}$

   Let x be the number of 2 point shots he makes and y be the number of 3 point shots he makes. We have $y \le x+1$ and $2x+3y = 23$ so $(x, y) = (10,1),(7,3),(4,5)$. We can imagine ordering our two and three point shots by choosing which slots the three point shots can go into. If there are x two points shots and y three point shots made, we have $\binom{x+1}{y}$ options, yielding the answer $\binom{11}{1}+\binom{8}{3}+\binom{5}{5} = \boxed{68}$ □

Performative

"To be performative is to be human," a wise matcha-sipping Labubu and baggy-jean-wearing softboi listening to Clairo with his iPhone 5 and dingy Delta Airlines earbuds once said while reading Infinite Jest in one hand and All About Love in the other.

1. **[6]** Clairo has x ounces of matcha, and Laufey has x ounces of horchata. Clairo fills up the rest of her drink up to 12 ounces with a 20% matcha, 80% horchata solution, while Laufey fills up the rest of her drink up to 8 ounces with a 80% matcha, 20% horchata solution. To their surprise, after following this procedure, each of their resulting drinks has the same concentration of matcha and horchata. Find x.

   Proposed by: Edwin Zhao

Solution. $\boxed{\dfrac{18}{5}}$

The concentration of matcha in each drink is $\frac{x+(12-x)\cdot 0.2}{12} = \frac{0.8(8-x)}{8}$. Solving the equation, we get $x = 3.6 = \boxed{\dfrac{18}{5}}$. $\square$

2. **[8]** Let L, A, U, F, E, Y be six (not necessarily distinct) digits such that $\overline{LAUFEY}$ is a prime number and $L+A+U+F+E+Y$ is an odd composite integer. Find the sum of all **three** possible values of $L+A+U+F+E+Y$.

   Proposed by: Atticus Oliver

   Solution. $\boxed{109}$

   The largest digit-sum of any six-digit number is $6\cdot 9 = 54$, thus only the odd composites less than 54 need be considered. Also note that if a number has a digit-sum divisible by 3, it is also divisible by 3, meaning that only the odd composites less than 54 also not divisible by 3 need to be considered. Notice that there are only three such values $(25, 35, 49)$, so the answer is simply their sum, or $\boxed{109}$. Note that $100699, 117899, 599899$ work for the numbers.

   $\square$

3. **[10]** Adam has 6 distinct feminist literature books in a tote bag. He draws a book randomly and puts it back into the tote bag. He stops when he gets 2 draws in a row of the same book, or 3 draws in a row each with different books. Find the expected number of times he must draw.

   Proposed by: Samuel Tsui

   Solution. $\boxed{3}$

   Note we want to find $\frac{5}{6}x+2$ where x is the expected number of rolls after rolling two distinct numbers as there is a $\frac{1}{6}$ chance he rolls the same number twice. We also have $x = \frac{1}{6}x+1$ since the only way he doesn't stop on the roll after two distinct rolls is if he get the same number he got two rolls ago. Thus $x = \frac{6}{5}$ so the answer is $\frac{5}{6}\cdot\frac{6}{5}+2 = \boxed{3}$. $\square$

4. **[12]** Let $ZIPS$ be a trapezoid with $ZI \parallel PS$ such that the following conditions are held:

   - if X is the midpoint of $\overline{SP}$, then $ZX = ZS$ and $\angle XZP = \angle PZI$,
   - and $\angle ISZ = \angle ISP$.

   Given that $ZI = 4$, find a quarter $[\triangle ZIP]$ (a quarter of the area of triangle ZIP).

   Proposed by: Ryan Tang

   Solution. $\boxed{\sqrt{3}}$

   First note that $\angle IZP = \angle ZPX = \angle XZP$ so $ZX = XP = ZS$. Furthermore, $\angle ZSI = \angle ISP = \angle ZIS$ so $ZS = IZ$. Thus, $IZ = XP$, but then this implies $ZIPX$ is a parallelogram, so $ZS = XZ = IP$, or $IPZS$ is isosceles trapezoid. Now, note that $XS = XP = ZX = ZS$, so ZSX is equilateral. Thus, IXP is also equilateral. We get the answer of $\frac{1}{4}\cdot\frac{4^2\sqrt{3}}{4} = \sqrt{3}$. $\square$

5. **[14]** A 4×4 grid is *performative* if each row, column, and 2×2 box in the corners contain exactly two T's, one O, and one E. Find the number of performative grids.

   Below is an example of a performative grid:

T	O	T	E
T	E	T	O
O	T	E	T
E	T	O	T

Proposed by: James Wu

Solution. $\boxed{112}$

Notice that if the constraints are satisfied for 'O' and 'E', the grid would be performative by putting 'T"s in the remaining cells, so we can ignore the fours completely.

There's 4 ways to place an 'O' in the top-left box. This fixes the row or column of the top-right and bottom-left boxes, leaving 2 ways for each, then the bottom-right box is forced. This yields $4 \times 2 \times 2 \times 1 = 16$ possibilities.

By symmetry, all arrangements of 'O"s are the same when considering the number of ways to place the threes. Therefore, we can use the following pattern to count (then multiply by 16):

O			
		O	
	O		
			O

We can perform casework on where we put the 3 in the top-left box:

- top-right: this fixes where the 'E' is in the top-right box, while there's 2 possibilities for the other two.
- bottom-left: same as top-right by symmetry
- bottom-right: there are two ways to place a 'E' in both the top-right and bottom-left boxes, but one of them forces the remaining three to occupy the same position as a 'O', so we have $4 - 1 = 3$ possibilities

Therefore, the answer is $16 \times (2+2+3) = \boxed{112}$ □

Tiebreaker Estimation

This problem will only be used to break ties for individual aggregate awards. If two competitors are tied, then the one closest to the correct answer will win.

1. **[TIEBREAKER]** Let $P(x) = (1+x)^{2025}$. Let c_k be the coefficient of the x^k term in $P(x)$. Let

$$c_{\max} = \max(c_0, c_1, \dots, c_{2025}).$$

Find the number of digits in $c_{\max}$.

Proposed by: Jerry Xu

Solution. $\boxed{607}$

$c_{\max}$ is the number of digits in $\binom{2025}{1012} = \frac{2025!}{1012!1013!}$, i.e.

$$c_{\max} = \left\lfloor \log_{10}\left(\frac{2025!}{1012!1013!}\right)\right\rfloor = \left\lfloor \log_{10}(2025!) - \log_{10}(1012!) - \log_{10}(1013!)\right\rfloor.$$

By Stirling's approximation,

$$\log_{10}(n!) \approx n\log_{10} n - n\log_{10} e.$$

Therefore,

$$\begin{aligned} c_{\max} &\approx (2025\log_{10} 2025 - 2025\log_{10} e) - (1012\log_{10} 1012 - 1012\log_{10} e) - (1013\log_{10} 1013 - 1013\log_{10} e) \\ &= 2025\log_{10} 2025 - 1012\log_{10} 1012 - 1013\log_{10} 1013. \end{aligned}$$

Note that $\log_{10} 1012 \approx \log_{10} 1013 \approx 3$, so the last two terms can be approximated as $-2025\cdot 3$. It is well-known that $\log_{10} 2 \approx 0.3$ is a very good approximation, so $\log_{10} 2025 \approx 3.3$. Then, our answer is

$$2025\cdot 3.2 - 2025\cdot 3 = 607.5.$$

The floor of this is $\boxed{607}$, which is exactly the true answer. □