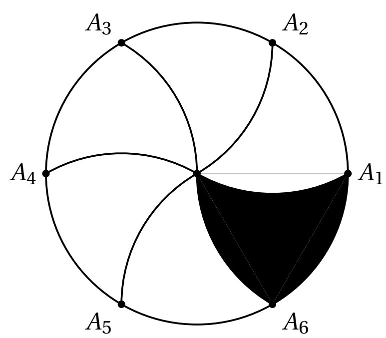
Speed Round

LMT Fall 2025

December 13th, 2025

1. **[6]** This year, 2025, is a special year because $2025 = 45^2$ is a perfect square. Find the next year after 2025 that is also a perfect square.

2. **[6]** The points A_1, A_2, A_3, A_4, A_5, and A_6 are equally spaced on the circumference of a unit circle. Find the area of the shaded region in the diagram.



3. **[6]** Alice and Bob, two LHS students, are trying to walk from Commons II to the Math building. Alice walks 4 meters north, then 9 meters west, then 8 meters north, at a speed of 1 meter per second. Bob departs at the same time as Alice, but walks in a straight line to the Math building at 0.5 meters per second. Find the absolute difference in their arrival times, in seconds.

4. **[6]** Jerry rolls two six-sided dice. The probability that the product of the two numbers that he rolled is even is p, and the probability that the sum of the two numbers that he rolled is even is q. Find $\frac{p}{q}$.

5. **[6]** Jame S. the window-washer is washing the windows of the new Lexington High School building. He works from 6AM to 10AM and he washes the windows from the 6th to the 10th floor. Find the average number of minutes it takes for him to wash one floor's windows.

6. **[6]** Let $\{a_n\}$ be a sequence such that $a_1 = 15, a_2 = 18$, and $a_n = a_{n-1} - a_{n-2}$ for all $n \geq 3$. Find the sum of all distinct terms in the sequence.

7. **[6]** Find the sum of all 2-digit numbers for which the sum of the tens digit, the ones digit, and the product of its 2 digits all sum to itself. For example, 19 satisfies these conditions because $1+9+1\cdot 9 = 19$.

8. **[6]** Define $a \star b$ to be $ab + a - b$. Find
$$(((2\star 3)\star 4)\star 5)\star 6.$$

9. **[6]** Let Ω be a circle with center O and radius 1. A semicircle ω with diameter AB is constructed such that A and B lie on circle O, O lies on ω, and O does not lie on line AB. Find the area of the region contained by the minor arc AB on Ω and arc AOB on ω.

10. **[6]** 6 distinct books are distributed among 3 students A, B, and C. Find the number of ways the books can be distributed such that one student gets 4 books, while the remaining students get 1 each.

11. **[6]** Arnab performs the following addition in base b:

$$\begin{array}{cccc} & S & S & 8_b \\ + & T & 5 & T_b \\ \hline 1 & 0 & 0 & 0_b \end{array}$$

where S, T, 5, and 8 are digits in base b. Find $S+T+b$ in base 10.

12. **[6]** The point $(3,4)$ is rotated around the point $(0,0)$ by some angle to point A. Point A is then rotated around the point $(3,0)$ by some angle to point B. Find the area of the region bounded by all possible B.

13. **[6]** In a group of 6 people, each pair of people is either friends or enemies. Find the number of ways for every person to have exactly 4 friends.

14. **[6]** Jerry chooses a random line that passes through the origin in the Cartesian plane. Find the probability that this line passes through the square with vertices at the points $(1,1,1)$, $(1,-1,1)$, $(-1,-1,1)$, and $(-1,1,1)$.

15. **[6]** On the algebra tryout, any real score between $[0,100]$ can be achieved. The average was 91, and the median was 100. Find the largest possible integer minimum of the grades.

16. **[6]** Find the number of subrectangles in a 6×6 grid that do not contain any cells in the middle 2×2.

17. **[6]** In isosceles trapezoid $ABCD$ with $AD \parallel BC$, the reflection of A across BD lies on line BC. Given that $AD=5, BC=7$, find the area of trapezoid $ABCD$.

18. **[6]** A candy is placed in one cell of a 4×4 square grid. Sam does not know where the candy is. In one move, Sam can check if the candy is within a selected 2×2 square grid. Find the least number of moves Sam must make to ensure he knows exactly which cell the candy is in.

19. **[6]** Find the number of up-right paths that start from the point $(0,0)$ and end when first reaching the line $y=5$ such that the path can't move in the same direction for more than 2 turns in a row.

20. **[6]** There exists a unique three-digit positive integer n that can be expressed in both $\overline{xxy}_{10}$ and $\overline{yxx}_b$, where $b<10$. Find n, in base 10.

21. **[6]** Let ABC be a triangle such a that $AB=3$, $BC=4$, $AC=5$. Let P be a point inside ABC such that $[PAB]+[PBC]=2$. Find the minimum possible value of BP.

22. **[6]** For $n\ge 2$, define $f(n)$ as the remainder of $1+2+\cdots+2n$ when divided by $1+2+\cdots+n$. Find $f(2)+f(3)+\cdots+f(14)$.

23. **[6]** Find the triple of positive reals (x,y,z) that satisfies:

$$xy = x+2y$$
$$yz = y+3z$$
$$zx = z+6x.$$

24. **[6]** Let ABC be a triangle with points P and Q on AB and AC respectively such that $BP=6$, $AP=7$, $CQ=7$, $AQ=5$. Let M be the midpoint of BC. If there exists a circle ω such that A, P, M, and Q all lie on the circumference of ω, find BC.

25. **[6]** Let a partition of a sequence be beautiful if the sequence is partitioned into contiguous blocks that are cyclic rotations of $[1,2,\cdots,k]$, where k is a positive integer. For example, $[2,3,1,1,2]$ is beautiful as it can be partitioned into $[2,3,1]$ and $[1,2]$. Find the sum of the number of beautiful partitions over all sequences of length 9.