## LMT Fall 2025 Guts Round - Part 1

Team Name: ______________________

______ 1. **[14]** Find the smallest prime number greater than 200.

______ 2. **[14]** Jerry selects 4 points on a plane. Find the maximum possible number of distinct triangles with vertices selected from Jerry's points.

______ 3. **[14]** Let $\overset{\frown}{AB}$ be a minor arc on the circle with measure $104°$. Let $X$ be a point on the circle distinct from $A$ and $B$. Find the positive difference between the maximum and minimum nonzero possible measures of $\angle AXB$.

## LMT Fall 2025 Guts Round - Part 2

Team Name: ______________________

______ 4. **[16]** Lonzo creates a three-digit passcode using the digits 1, 2, 3, and 4. Find the number of passcodes that have an even sum of digits, where digits are not necessarily distinct.

______ 5. **[16]** On a planet there are three groups of aliens: "Jerries", "Bens", and "Peters". One day, at a meeting, 4 Jerries, 8 Bens, and 10 Peters show up. Given that each alien shakes hands with every other alien not from their own group, find the number of total number of handshakes that occur.

______ 6. **[16]** Lebron and Jordan both take $x$ free throw attempts, with Lebron making 50% of them while Jordan makes 0%. Then, Lebron shoots $2y$ more attempts and makes 60% of them, while Jordan shoots $3y$ more attempts and also makes 60%. Given they end up making the same percentage of all of their shots, find the minimum possible value of $x + y$.

## LMT Fall 2025 Guts Round - Part 3

Team Name: ______________________

______ 7. **[18]** Let $ABCD$ be a parallelogram and $M$ the midpoint of $CD$. Given that $AM = 3, BM = 4, AB = 5$, find $BC$.

______ 8. **[18]** There exist real numbers $a$ and $b$ such that $a + \frac{1}{b} = 7$ and $b + \frac{1}{a} = 6$. Find the sum of all possible values of $ab$.

______ 9. **[18]** Today, 12/13/25, has a special property: The sum of the month (12) and the day (13) is equal to the last two digits of the year (25). Find the number of distinct dates in the 21st century with this property.

## LMT Fall 2025 Guts Round - Part 4

Team Name: ______________________

______ 10. **[20]** Find the number of rearrangements of the word *MATHTEAM* that contain either the substring *MATH* or the substring *HEAT*.

______ 11. **[20]** Find the largest prime factor of 142857.

______ 12. **[20]** In triangle $ABC, \angle A = 72°, \angle B = 68°, \angle C = 40°$. Let the incenter of $\triangle ABC$ be $I$ and let $\overline{AI}$ intersect $BC$ at point $D$. The tangent from $D$ to the incircle of $ABC$ intersects $\overline{AC}$ at point $E$. Find $\angle IEC$.

## LMT Fall 2025 Guts Round - Part 5

Team Name: ______________________________

______________ 13. **[22]** Let $TNAG$ be a square where $\overline{GT} = 6$. A point $C$ is chosen so that $\overline{CT} \le 12$. Given that quadrilateral $CNAG$ is convex, find the area of the region bounded by all possible $C$.

______________ 14. **[22]** Find the number of ways to color each cell of $4 \times 4$ grid red or blue such that there are 8 cells of each color and each row and column has at least 3 cells of the same color.

______________ 15. **[22]** Let $a$ and $b$ be integers with $a$ greater than $b$ satisfying $a^2b^2 + 2ab + 9 = a^2 + b^2$. Find all possible values of $(a, b)$

## LMT Fall 2025 Guts Round - Part 6

Team Name: ______________________________

______________ 16. **[24]** Real numbers $a, b, c, d$ satisfy $a^2 + b^2 = 4$ and $(c-5)^2 + (d-12)^2 = 9$. Find the minimum value of $(c-a)^2 + (d-b)^2$.

______________ 17. **[24]** Ethan and 2 girls are placed independently and uniformly at random on the circumference of a circle of radius 1. Ethan can attract any girl within a distance of 1, but the girls are jealous of each other if the distance between them is at most 1. Find the probability that Ethan can attract both girls without making them jealous.

______________ 18. **[24]** Let $a > b > c > 0$ be real numbers satisfying

$$(a-b)(a-c) + 7 = b^2 + c^2$$

$$(b-a)(b-c) + 31 = a^2 + c^2$$

$$(c-a)(c-b) + 31 = a^2 + b^2$$

Find $(a, b, c)$.

## LMT Fall 2025 Guts Round - Part 7

Team Name: ______________________________

______________ 19. **[26]** Awashonks is graphing the function

$$f(x) = \left\lfloor \frac{5}{3}x \right\rfloor + \left\lceil \frac{5}{9}x^2 \right\rceil + \left\lfloor \frac{5}{27}x^3 \right\rfloor + \left\lceil \frac{5}{81}x^4 \right\rceil$$

from $x = 0$ to $x = 6$. Find the minimum distance her pen must travel, considering any jumps it must take.

______________ 20. **[26]** Let $ABC$ be a triangle. From each midpoint of an edge of $ABC$ draw an altitude to each of the two sides that it does not lie on. This forms a hexagon in the interior of $ABC$. Given that $AB = 5$, $BC = 6$, and $AC = 7$, find the area of this hexagon.

______________ 21. **[26]** Let $(a, b, c)$ be a ordered triple of positive reals that satisfy

$$\lfloor ab \rfloor c = 3$$

$$\lfloor bc \rfloor a = 4$$

$$\lfloor ca \rfloor b = 5.$$

Find the sum of all possible values of $a$.

## LMT Fall 2025 Guts Round - Part 8

Team Name: ______________________________

______________ 22. **[30]** Let $ABCD$ be a trapezoid such that $AB \parallel CD$ and $DA = AB = BC = 5$. Let $EFGH$ be a square of side length 1 inside $ABCD$ such that $GH$ lies on $CD$ with $G$ in between $C$ and $H$. If $BF = 5$ and $CG = 7$, find the area of $ADE$.

______________ 23. **[30]** On Alfredo's strange clock, the minute and hour hand are identical. During a 12 hour period, find the number of times for which it is impossible for Alfredo to tell the real time. (For example, if the two hands are pointed at 12 and 3, Alfredo can tell that the real time is 3:00.)

______________ 24. **[30]** For an integer with at least three digits define its *smae* to be the integer formed by swapping the second and third digit. For example, the smae of 12345 is 13245. Let $n$ be a 5-digit positive integer, and let $m$ be its smae. Suppose $n$ is the unique such 5-digit integer whose prime factorization is the first three digits of $m$ times the last three digits of $m$. Find $n$.

## LMT Fall 2025 Guts Round - Part 9

Team Name: ______________________________

______________ 25. **[30]** Find the number of primes $p$ which can be written in the form $2^n + 2^m + 1$ for positive integers $m \le n < 100$. If your answer is $A$ and the correct answer is $E$, then you will earn $\max(0, \lceil 30 - 0.45|E - A|\rceil)$ points.

______________ 26. **[30]** The numbers $1, 2, \ldots, 10$ are placed in a line in that order. Then, for each of the nine pairs of consecutive integers, an operation is chosen independently at uniformly at random from $+, -, \cdot, \div$ and placed between those integers. For example, one possible outcome for the resulting expression is

$$1 + 2 \cdot 3 + 4 \div 5 \cdot 6 \cdot 7 + 8 - 9 \div 10.$$

Find the expected value of the resulting expression. If the true answer is $A$ and your answer is $E$, you will earn $\max(0, \lceil 30 - 0.45|E - A|^2\rceil)$ points.

______________ 27. **[30]** $N$ points are chosen independently and uniformly at random on a circle of radius 1. Find the smallest positive integer $N$ such that the expected value of the area of the $N$-gon formed by those $N$ points is at least 3.14. If the correct answer is $E$ and your answer is $A$, you will earn $\max(0, \lceil 30 - 0.45|E - A|\rceil)$ points.