## Accuracy Round

## Lexington High School

May 14th, 2022

1. [6] Kevin colors a ninja star on a piece of graph paper where each small square has area 1 square inch. Find the area of the region colored, in square inches.

2. [8] Let $a \boldsymbol{\wedge} b=\frac{a^{2}-b^{2}}{2 b-2 a}$. Given that $3 \boldsymbol{\propto} x=-10$, compute $x$.
3. [10] Find the difference between the greatest and least values of $\operatorname{lcm}(a, b, c)$, where $a, b$, and $c$ are distinct positive integers between 1 and 10, inclusive.
4. [12] Kevin runs uphill at a speed that is 4 meters per second slower than his speed when he runs downhill. Kevin takes a total of 80 seconds to run up and down a hill on one path. Given that the path is 300 meters long (he travels 600 meters total), find how long Kevin takes to run up the hill in seconds.
5. [14] A bag contains 5 identical blue marbles and 5 identical green marbles. In how many ways can 5 marbles from the bag be arranged in a row if each blue marble must be adjacent to at least 1 green marble?
6. [16] Jacob likes to watch Mickey Mouse Clubhouse! One day, he decides to create his own Mickey Mouse head shown below, with two circles $\omega_{1}$ and $\omega_{2}$ and a circle $\omega$, and centers $O_{1}, O_{2}$, and $O$, respectively. Let $\omega_{1}$ and $\omega$ meet at points $P_{1}$ and $Q_{1}$, and let $\omega_{2}$ and $\omega$ meet at points $P_{2}$ and $Q_{2}$. Point $P_{1}$ is closer to $O_{2}$ than $Q_{1}$, and point $P_{2}$ is closer to $O_{1}$ than $Q_{2}$. Given that $P_{1}$ and $P_{2}$ lie on $O_{1} O_{2}$ such that $O_{1} P_{1}=P_{1} P_{2}=P_{2} O_{2}=2$, and $Q_{1} O_{1} \| Q_{2} O_{2}$, the area of $\omega$ can be written as $n \pi$. Find $n$.

7. [18] A teacher wishes to separate her 12 students into groups. Yesterday, the teacher put the students into 4 groups of 3 . Today, the teacher decides to put the students into 4 groups of 3 again. However, she doesn't want any pair of students to be in the same group on both days. Find how many ways she could form the groups today.
8. [20] A ray originating at point $P$ intersects a circle with center $O$ at points $A$ and $B$, with $P B>P A$. Segment $\overline{O P}$ intersects the circle at point $C$. Given that $P A=31, P C=17$, and $\angle P B O=60^{\circ}$, find the radius of the circle.
9. [22] A rook is randomly placed on an otherwise empty $8 \times 8$ chessboard. Owen makes moves with the rook by randomly choosing 1 of the 14 possible moves. Find the expected value of the number of moves it takes Owen to move the rook to the top left square. Note that a rook can move any number of squares either in the horizontal or vertical direction each move.
10. [24] In a room, there are 100 light switches, labeled with the positive integers $\{1,2, \ldots, 100\}$. They're all initially turned off. On the $i$ th day for $1 \leq i \leq 100$, Bob flips every light switch with label number $k$ divisible by $i$ a total of $\frac{k}{i}$ times. Find the sum of the labels of the light switches that are turned on at the end of the 100th day.
11. [TIEBREAKER] Let $L$ be the number of times the letter $L$ appeared on the Speed Round, $M$ be the number of times the letter $M$ appeared on the Speed Round, and $T$ be the number of times the letter $T$ appeared on the Speed Round. Find the value of $L M T$.
