Accuracy Round

Lexington High School

May 14th, 2022

1. [6] Kevin colors a ninja star on a piece of graph paper where each small square has area 1 square inch. Find the area of the region colored, in square inches.



- 2. [8] Let $a \triangleq b = \frac{a^2 b^2}{2b 2a}$. Given that $3 \triangleq x = -10$, compute *x*.
- 3. [10] Find the difference between the greatest and least values of lcm(*a*, *b*, *c*), where *a*, *b*, and *c* are distinct positive integers between 1 and 10, inclusive.
- 4. **[12]** Kevin runs uphill at a speed that is 4 meters per second slower than his speed when he runs downhill. Kevin takes a total of 80 seconds to run up and down a hill on one path. Given that the path is 300 meters long (he travels 600 meters total), find how long Kevin takes to run up the hill in seconds.
- 5. **[14]** A bag contains 5 identical blue marbles and 5 identical green marbles. In how many ways can 5 marbles from the bag be arranged in a row if each blue marble must be adjacent to at least 1 green marble?
- 6. **[16]** Jacob likes to watch Mickey Mouse Clubhouse! One day, he decides to create his own Mickey Mouse head shown below, with two circles ω_1 and ω_2 and a circle ω , and centers O_1 , O_2 , and O, respectively. Let ω_1 and ω meet at points P_1 and Q_1 , and let ω_2 and ω meet at points P_2 and Q_2 . Point P_1 is closer to O_2 than Q_1 , and point P_2 is closer to O_1 than Q_2 . Given that P_1 and P_2 lie on O_1O_2 such that $O_1P_1 = P_1P_2 = P_2O_2 = 2$, and $Q_1O_1 \parallel Q_2O_2$, the area of ω can be written as $n\pi$. Find n.



7. **[18]** A teacher wishes to separate her 12 students into groups. Yesterday, the teacher put the students into 4 groups of 3. Today, the teacher decides to put the students into 4 groups of 3 again. However, she doesn't want any pair of students to be in the same group on both days. Find how many ways she could form the groups today.

- 8. [20] A ray originating at point *P* intersects a circle with center *O* at points *A* and *B*, with PB > PA. Segment \overline{OP} intersects the circle at point *C*. Given that PA = 31, PC = 17, and $\angle PBO = 60^{\circ}$, find the radius of the circle.
- 9. **[22]** A rook is randomly placed on an otherwise empty 8 × 8 chessboard. Owen makes moves with the rook by randomly choosing 1 of the 14 possible moves. Find the expected value of the number of moves it takes Owen to move the rook to the top left square. Note that a rook can move any number of squares either in the horizontal or vertical direction each move.
- 10. [24] In a room, there are 100 light switches, labeled with the positive integers $\{1, 2, ..., 100\}$. They're all initially turned off. On the *i*th day for $1 \le i \le 100$, Bob flips every light switch with label number *k* divisible by *i* a total of $\frac{k}{i}$ times. Find the sum of the labels of the light switches that are turned on at the end of the 100th day.
- 11. **[TIEBREAKER]** Let *L* be the number of times the letter *L* appeared on the Speed Round, *M* be the number of times the letter *M* appeared on the Speed Round, and *T* be the number of times the letter *T* appeared on the Speed Round. Find the value of *LMT*.