LMT "Fall" Guts Round - Part 1

Team Name: ______________________________

__________ 1. **[4]** Ephram was born in May 2005. How old will he turn in the first year where the product of the digits of the year number is a nonzero perfect square?

__________ 2. **[4]** Zhao is studying for his upcoming calculus test by reviewing each of the 13 lectures, numbered Lecture 1, Lecture 2, ... Lecture 13. For each n, he spends $5n$ minutes on Lecture n. Which lecture is he reviewing after 4 hours?

__________ 3. **[4]** Compute

$$\frac{3^3 \div 3(3)+3}{\frac{3}{3}}+3!.$$

LMT "Fall" Guts Round - Part 2

Team Name: ______________________________

__________ 4. **[5]** At Ingo's shop, train tickets normally cost \$2, but every 5th ticket costs only \$1. At Emmet's shop, train tickets normally cost \$3, but every 5th ticket is free. Both Ingo and Emmett sold 1000 tickets. Find the absolute difference between their sales, in dollars.

__________ 5. **[5]** Ephram paddles his boat in a river with a 4-mph current. Ephram travels at 10 mph in still water. He paddles downstream and then turns around and paddles upstream back to his starting position. Find the proportion of time he spends traveling upstream, as a percentage.

__________ 6. **[5]** The average angle measure of a 13-14-15 triangle is m° and the average angle measure of a 5-6-7 triangle is n°. Find $m-n$.

LMT "Fall" Guts Round - Part 3

Team Name: ______________________________

__________ 7. **[6]** Let $p(x)=x^2-10x+31$. Find the minimum value of $p(p(x))$ over all real x.

__________ 8. **[6]** Michael H. and Michael Y. are playing a game with 4 jellybeans. Michael H starts with 3 of the jellybeans, and Michael Y starts with the remaining 1. Every minute, a Michael flips a coin, and if heads, Michael H takes a jellybean from Michael Y. If tails, Michael Y takes a jellybean from Michael H. Whichever Michael gathers all 4 jellybeans wins. The probability Michael H wins can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A+B$.

__________ 9. **[6]** Define the *digit-product* of a positive integer to be the product of its non-zero digits. Let M denote the greatest five-digit number with a *digit-product* of 360, and let N denote the least five-digit number with a *digit-product* of 360. Find the *digit-product* of $M-N$.

LMT "Fall" Guts Round - Part 4

Team Name: ____________________

________ 10. **[7]** Hannah is attending one of the three IdeaMath classes running at LHS, while Alex decides to randomly visit some combination of classes. He won't visit all three classes, but he's equally likely to visit any other combination. The probability Alex visits Hannah's class can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A+B$.

________ 11. **[7]** In rectangle $ABCD$, let E be the intersection of diagonal AC and the circle centered at A passing through D. Angle $\angle ACD = 24°$. Find the measure of $\angle CED$ in degrees.

________ 12. **[7]** During his IdeaMath class, Zach writes the numbers 2, 3, 4, 5, 6, 7, and 8 on a whiteboard. Every minute, he chooses two numbers a and b from the board, erases them, and writes the number $ab+a+b$ on the board. He repeats this process until there's only one number left. Find the sum of all possible remaining numbers.

LMT "Fall" Guts Round - Part 5

Team Name: ____________________

________ 13. **[9]** In isosceles right $\triangle ABC$ with hypotenuse AC, Let A' be the point on the extension of AB past A such that $AA' = 1$. Let C' be the point on the extension of BC past vertex C such that $CC' = 2$. Given that the difference of the areas of triangle $A'BC'$ and ABC is 10, find the area of ABC.

________ 14. **[9]** Compute the sum of the greatest and least values of x such that

$$(x^2-4x+4)^2+x^2-4x \le 16.$$

________ 15. **[9]** Ephram is starting a fan club. At the fan club's first meeting, everyone shakes hands with everyone else exactly once, except for Ephram, who is extremely sociable and shakes hands with everyone else twice. Given that a total of 2015 handshakes took place, how many people attended the club's first meeting?

LMT "Fall" Guts Round - Part 6

Team Name: ____________________

________ 16. **[11]** Let a be a solution to $x^3-x+1=0$. Find a^6-a^2+2a.

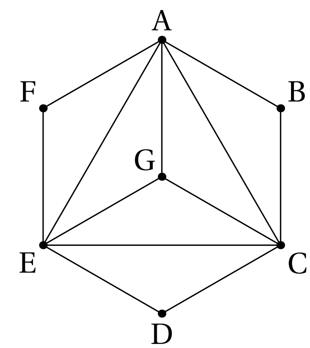
________ 17. **[11]** For a positive integer n, $\phi(n)$ is the number of positive integers less than n that are relatively prime to n. Compute the sum of all n for which $\phi(n) = 24$.

________ 18. **[11]** Let x be a positive integer such that $x^2 \equiv 57 \pmod{59}$. Find the least possible value of x.

LMT "Fall" Guts Round - Part 7

Team Name: ______________________________

______ 19. **[13]** In the diagram below, find the number of ways to color each vertex red, green, yellow or blue such that no two vertices of a triangle have the same color.



______ 20. **[13]** In a set with n elements, the sum of the number of ways to choose 3 or 4 elements is a multiple of the sum of the number of ways to choose 1 or 2 elements. Find the number of possible values of n between 4 and 120 inclusive.

______ 21. **[13]** In unit square $ABCD$, let Γ be the locus of points P in the interior of $ABCD$ such that $2AP < BP$. The area of Γ can be written as $\frac{a\pi + b\sqrt{c}}{d}$ for integers a, b, c, d with c squarefree and $\gcd(a, b, d) = 1$. Find $1000000a + 10000b + 100c + d$.

LMT "Fall" Guts Round - Part 8

Team Name: ______________________________

______ 22. **[15]** Ephram, GammaZero, and Orz walk into a bar. Each write some permutation of the letters "LMT" once, then concatenate their permutations one after the other (i.e. LTMTLMTLM would be a possible string, but not LLLMMMTTT). Suppose that the probability that the string "LMT" appears in that order among the new 9-character string can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.

______ 23. **[15]** In $\triangle ABC$ with side lengths $AB = 27$, $BC = 35$, and $CA = 32$, let D be the point at which the incircle is tangent to BC. The value of $\frac{\sin\angle CAD}{\sin\angle BAD}$ can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.

______ 24. **[15]** Let A be the greatest possible area of a square contained in a regular hexagon with side length 1. Let B be the least possible area of a square that contains a regular hexagon with side length 1. The value of $B - A$ can be expressed as $a\sqrt{b} - c$ for positive integers a, b, and c with b squarefree. Find $10000a + 100b + c$.

LMT "Fall" Guts Round - Part 9

Team Name: ______________________________

______________ 25. **[10]** Estimate how many days before today this problem was written. If your estimation is E and the actual answer is A, you will receive $\max\left(\left\lfloor 10-\left|\frac{E-A}{2}\right|\right\rfloor, 0\right)$ points.

______________ 26. **[10]** Circle ω_1 is inscribed in unit square $ABCD$. For every integer $1 < n \le 10,000$, ω_n is defined as the largest circle which can be drawn inside $ABCD$ that does not overlap the interior of any of $\omega_1, \omega_2, \ldots, \omega_{n-1}$ (If there are multiple such ω_n that can be drawn, one is chosen at random). Let r be the radius of $\omega_{10,000}$. Estimate $\frac{1}{r}$. If your estimation is E and the actual answer is A, you will receive $\max\left(\left\lfloor 10-\left|\frac{E-A}{200}\right|\right\rfloor, 0\right)$ points.

______________ 27. **[?]** Answer with a positive integer less than or equal to 20. We will compare your response with the response of every other team that answered this problem. When two equal responses are compared, neither team wins. When two unequal responses $A > B$ are compared, A wins if $B \mid A$, and B wins otherwise. If your team wins n times, you will receive $\left\lfloor \frac{n}{2} \right\rfloor$ points.