1. **[10]** Given that the expression
$$\frac{20^{21}}{20^{20}} + \frac{20^{20}}{20^{21}}$$
can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

Proposed by Ada Tsui

Solution. $\boxed{421}$

Simplifying, we have
$$20 + \frac{1}{20} = \frac{400}{20} + \frac{1}{20} = \frac{401}{20}.$$
Then $m = 401, n = 20$, and $m + n = \boxed{421}$. □

2. **[10]** Find the greatest possible distance between any two points inside or along the perimeter of an equilateral triangle with side length 2.

Proposed by Alex Li

Solution. $\boxed{2}$

The greatest possible distance is between two vertices, which is just equal to the side length of $\boxed{2}$. □

3. **[10]** Aidan rolls a pair of fair, six sided dice. Let n be the probability that the product of the two numbers at the top is prime. Given that n can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a+b$.

Proposed by Aidan Duncan

Solution. $\boxed{7}$

One of the numbers must be 1, and the other one must be 2, 3, or 5. There are 6 ways to do this, so the probability is $\frac{6}{6\cdot 6} = \frac{1}{6}$, so $a + b = 7$. □

4. **[10]** Set S contains exactly 36 elements in the form of $2^m \cdot 5^n$ for integers $0 \le m, n \le 5$. Two distinct elements of S are randomly chosen. Given that the probability that their product is divisible by 10^7 is $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a+b$.

Proposed by Ada Tsui

Solution. $\boxed{349}$

For the product to be divisible by 10^7, the exponent of the 2 and the 5 in the product must both be greater than or equal to 7, or the exponents of the 2 and 5 in the elements must individually sum to be greater than or equal to 7.

The number of ways for the exponents of the 2 in the elements to sum to a number greater than or equal to 7 is 10. (There are two ways from each of $(5,4),(5,3),(5,2),(4,3)$ and one way from each of $(5,5),(4,4)$.

The number of ways for the exponents of 5 in the elements to sum to a number greater than or equal to 7 is the same as the number of ways for the exponents of 2, 10. So there are $10 \cdot 10 = 100$ ways for the product to be divisible by 10^7.

But the exponents of 2 and 5 cannot be exactly the same because then the elements would be exactly the same (and the elements need to be distinct). So we must subtract out 4 of the ways (since each of the exponents can overlap with the pairs $(5,5),(4,4)$). So there are $100 - 4 = 96$ favorable outcomes.

Of course, the number of total outcomes is $36 \cdot 35$. (There are 6 outcomes for the first exponent and 6 outcomes for the second exponent, and one less outcome for the second element.) Then the probability is $\frac{96}{36\cdot 35} = \frac{8}{105}$

Thus, $a = 8, b = 105$ and $a + b = \boxed{113}$ □

5. **[15]** Find the number of ways there are to permute the elements of the set $\{1,2,3,4,5,6,7,8,9\}$ such that no two adjacent numbers are both even or both odd.

Proposed by Ephram Chun

Solution. $\boxed{2880}$

Simply alternating starting with an odd number. Thus our answer is $5!4! = \boxed{2880}$. □

6. **[15]** Maisy is at the origin of the coordinate plane. On her first step, she moves 1 unit up. On her second step, she moves 1 unit to the right. On her third step, she moves 2 units up. On her fourth step, she moves 2 units to the right. She repeats this pattern with each odd-numbered step being 1 unit more than the previous step. Given that the point that Maisy lands on after her 21st step can be written in the form (x, y), find the value of $x + y$.

   Proposed by Audrey Chun

   Solution. $\boxed{121}$

   To find the 21st step, we can find the 20th step first and then add 11 to the y value. The point after each even step will have the same x and y value so we can add up the horizontal steps to get our x value and then put the same number for the y value. $1+2+3+4+5+6+7+8+9+10 = 55$ so our point after the 20th step is $(55,55)$ and the point after the 21st step is $(55,66)$. Therefore, our answer is $\boxed{121}$. □

7. **[15]** Given that x and y are positive real numbers such that

$$\frac{5}{x} = \frac{y}{13} = \frac{x}{y},$$

   find the value of $x^3 + y^3$.

   Proposed by Ephram Chun

   Solution. $\boxed{1170}$

$$\frac{5^2}{x^2} = \frac{x}{13} \implies x^3 = 5^2 \cdot 13 = 325.$$

   Similarly,

$$\frac{5}{y} = \frac{y^2}{13^2} \implies y^3 = 13^2 \cdot 5 = 845.$$

   So $x^3 + y^3 = 325 + 845 = \boxed{1170}$. □

8. **[15]** Find the number of arithmetic sequences a_1, a_2, a_3 of three nonzero integers such that the sum of the terms in the sequence is equal to the product of the terms in the sequence.

   Proposed by Sammy Charney

   Solution. $\boxed{4}$

   Let the middle term be b and the common difference be d. The sum of the terms is $3b$ and the product is $b(b^2 - d^2)$. Thus, $b^2 - d^2 = 3$, so $|b| = 2$ and $|d| = 1$, giving us $\boxed{4}$ solutions. □

9. **[20]** Convex pentagon $PQRST$ has $PQ = TP = 5$, $QR = RS = ST = 6$, and $\angle QRS = \angle RST = 90°$. Given that points U and V exist such that $RU = UV = VS = 2$, find the area of pentagon $PQUVT$.

   Proposed by Kira Tang

   Solution. $\boxed{36}$

   Triangle AMG is an isosceles triangle with congruent sides $AM = GA = 5$ and base $MG = 6$. Drawing the perpendicular bisector of MG, we see that triangle AMG has height 4. We also find that U and S lie on ON, and that quadrilateral $MONG$ is a square. Thus pentagon $AMUSG$ is the resulting area when the areas of triangles MOU and SNG are subtracted from the area of pentagon $AMONG$. $AMONG$ has an area that is the sum of the areas of AMG and $MONG$. Thus the area of $AMUSG$ is $12 + 36 - 6 - 6 = \boxed{36}$. □

10. **[20]** Let $f(x)$ be a function mapping real numbers to real numbers. Given that $f(f(x)) = \frac{1}{3x}$, and $f(2) = \frac{1}{9}$, find $f\left(\frac{1}{6}\right)$.

    Proposed by Zachary Perry

Solution. $\boxed{3}$

Note that $f(f(f(x))) = f\left(\frac{1}{3x}\right) = \frac{1}{3f(x)}$. Plugging in $x = 2$ gives the answer of $\boxed{3}$. $\square$

11. **[20]** In rectangle $ABCD$, points E and F are on sides $\overline{BC}$ and $\overline{AD}$, respectively. Two congruent semicircles are drawn with centers E and F such that they both lie entirely on or inside the rectangle, the semicircle with center E passes through C, and the semicircle with center F passes through A. Given that $AB = 8$, $CE = 5$, and the semicircles are tangent, find the length BC.

Proposed by Ada Tsui

Solution. $\boxed{16}$

Let $BC = x$ and G be a point on BC such that FG is perpendicular to BC. Then $FG = AB = 8$ and $BG = CE = 5$, so $EG = x - 10$.

Connect points E, F, G such that the right triangle EFG is formed. Then EF is twice the shared radii of the semicircles, $2CE = 10$.

As EFG is a right triangle with the values of EF and FG, the value of EG can be found with the Pythagorean Theorem. That is, $EG = 6$. Of course, EG is also $x - 10$, so $x - 10 = 6$ and $x = 16$.

Thus, $x = 16$ and $BC = 16$. $\square$

12. **[20]** Given that the expected amount of 1s in a randomly selected 2021-digit number is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m + n$.

Proposed by Hannah Shen

Solution. $\boxed{1828}$

We can consider the sum of the probability that each individual digit is 1. The leftmost digit can be any of the 9 integers from 1 to 9 inclusive, so we expect the digit to be 1 with a $\frac{1}{9}$ chance. Each of the 2020 remaining digits can be any of the 10 integers from 0 to 9 inclusive, so we expect each of these to be 1 with a $\frac{1}{10}$ chance. Our final answer is $\frac{1}{9} + 2020 * \frac{1}{10} = \frac{1819}{9} \implies \boxed{1828}$. $\square$

13. **[25]** Call a 4-digit number $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$ *unnoticeable* if $a + c = b + d$ and $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d} + \underline{c}\,\underline{d}\,\underline{a}\,\underline{b}$ is a multiple of 7. Find the number of unnoticeable numbers.
Note: a, b, c, and d are nonzero distinct digits.

Proposed by Aditya Rao

Solution. $\boxed{32}$

Say $a + c = b + d = x$. Therefore, $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d} + \underline{c}\,\underline{d}\,\underline{a}\,\underline{b} = 1000x + 100x + 10x + x = 1111x = 101 * 11 * x$. Since 101 and 11 are prime, x has to be a multiple of 7 in order for $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d} + \underline{c}\,\underline{d}\,\underline{a}\,\underline{b}$ to be a multiple of 7. So, either $a + c = b + d = 7$, or $a + c = b + d = 14$. There are 24 ways to make the first equation true, while there are 8 ways to make the second equation true. Therefore, there are $\boxed{32}$ total "Unnoticeable" numbers. I told you they were unnoticeable! $\square$

14. **[25]** In the expansion of

$$(2x + 3y)^{20},$$

find the number of coefficients divisible by 144.

Proposed by Hannah Shen

Solution. $\boxed{16}$

144 can be rewritten as $2^4 3^2$. It is evident that all 15 terms from $\binom{20}{4}(2x)^4(3y)^{16}$ to $\binom{20}{18}(2x)^{18}(3x)^2$ have coefficients divisible by 144. Of the remaining 6 terms, only $\binom{20}{3}(2x)^3(3x)^{17}$ has a coefficient divisible by 144, since $\binom{20}{3}$ contributes a factor of 2. This gives a total of $\boxed{16}$ terms. $\square$

15. **[25]** A geometric sequence consists of 11 terms. The arithmetic mean of the first 6 terms is 63, and the arithmetic mean of the last 6 terms is 2016. Find the 7th term in the sequence.

Proposed by Powell Zhang

Solution. $\boxed{384}$

$\frac{2016}{63} = 32 = 2^5$. Since of the last six terms, each term is five terms later than its corresponding term in the first six terms. Thus we know that the common ratio of the geometric sequence is 2. We know that the sum of the first 6 terms is $t_1(2^6-1) = 6\cdot 63 = 378 \implies t_1 = 6$ so we know the first term is 6. Thus, the 7th term is just $6\cdot 2^6 = 384$. □

16. **[25]** Bob plants two saplings. Each day, each sapling has a $\frac{1}{3}$ chance of instantly turning into a tree. Given that the expected number of days it takes both trees to grow is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.

Proposed by Powell Zhang

Solution. $\boxed{26}$

Let a_n be the expected number of days it takes all trees to grow when n trees have already grown. This gives us:

$$a_0 = 1 + \frac{4}{9}a_0 + \frac{4}{9}a_1 + \frac{1}{9}a_2$$

$$a_1 = 1 + \frac{2}{3}a_1 + \frac{1}{3}a_2$$

$$a_2 = 0$$

Solving this gives us

$$a_1 = 3$$

and

$$a_0 = \frac{21}{5} \implies \boxed{26}.$$

□

17. **[30]** In $\triangle ABC$ with $\angle BAC = 60^\circ$ and circumcircle ω, the angle bisector of $\angle BAC$ intersects side $\overline{BC}$ at point D, and line AD is extended past D to a point A'. Let points E and F be the feet of the perpendiculars of A' onto lines AB and AC, respectively. Suppose that ω is tangent to line EF at a point P between E and F such that $\frac{EP}{FP} = \frac{1}{2}$. Given that $EF = 6$, the area of $\triangle ABC$ can be written as $\frac{m\sqrt{n}}{p}$, where m and p are relatively prime positive integers, and n is a positive integer not divisible by the square of any prime. Find $m+n+p$.

Proposed by Taiki Aiba

Solution. $\boxed{52}$

Note that quadrilateral $AEA'F$ is two $30-60-90$ right triangles AEA' and AFA' glued together by their hypotenuses $\overline{AA'}$. Thus, we have that $\triangle AEF$ is equilateral. Next, we will use Power of a Point with $\triangle AEF$ and ω. We see that $EP = 2$ and $FP = 4$. Note that line AE intersects ω at B and A, and line AF intersects ω at C and A. We have the equations

$$2^2 = BE(6) \quad \text{and} \quad 4^2 = CF(6).$$

Note that $AB = AE - BE = 6 - BE$ and $AC = AF - CF = 6 - CF$, so we get

$$4 = (6 - AB)6 \quad \text{and} \quad 16 = (6 - AC)6.$$

Solving gives us $AB = \frac{16}{3}$ and $AC = \frac{10}{3}$. Finally, we have that the area of $\triangle ABC$ is

$$\frac{16}{3}\cdot\frac{10}{3}\cdot\sin(60^\circ)\cdot\frac{1}{2} = \frac{40\sqrt{3}}{9},$$

for an answer of $40+3+9 = \boxed{52}$. □

18. **[30]** There are 23 balls on a table, all of which are either red or blue, such that the probability that there are n red balls and $23-n$ blue balls on the table ($1 \le n \le 22$) is proportional to n. (e.g. the probability that there are 2 red balls and 21 blue balls is twice the probability that there are 1 red ball and 22 blue balls.) Given that the probability that the red balls and blue balls can be arranged in a line such that there is a blue ball on each end, no two red balls are next to each other, and an equal number of blue balls can be placed between each pair of adjacent red balls is $\frac{a}{b}$, where a and b are relatively prime positive integers, find $a+b$.
Note: There can be any nonzero number of consecutive blue balls at the ends of the line.
Proposed by Ada Tsui

Solution. $\boxed{29}$

Let the number of blue balls be denoted by b. We have the requirement, first, that there be at least two blue balls to fill in the ends. Next, we just need that

$$b-2 \ge k(23-b)-k \implies (22-b)(k+1) \le 20 \implies b \ge 12,$$

so that we can insert k blue balls in between every pair of adjacent red balls, and then put the leftover blue balls in random places, just not between any red balls. Now, our probability is

$$\frac{1+2+3+\cdots+11}{1+2+3+\cdots+22} = \frac{6}{23} \implies \boxed{29}.$$

□

19. **[30]** Kevin is at the point $(19,12)$. He wants to walk to a point on the ellipse $9x^2+25y^2=8100$, and then walk to $(-24,0)$. Find the shortest length that he has to walk.
Proposed by Kevin Zhao

Solution. $\boxed{47}$

Note that the foci of the ellipse are $(-24,0)$ and $(24,0)$ and that the sum of the distances from any point on the ellipse to a focal point is going to be twice the major axis, which is doing to be $2\cdot 30 = 60$ in this case. So, since the shortest path between two points is a straight line, then we see that if Kevin goes from $(24,0)$ to $(19,12)$ to the point on the ellipse (all on a straight line) and then to $(-24,0)$, then he will walk 60 units. As a result, our answer will be 60 units minus the distance from $(24,0)$ to $(19,12)$. The distance from $(24,0)$ to $(19,12)$ is going to be $\sqrt{(24-19)^2+(0-12)^2} = \sqrt{5^2+12^2} = 13$, so our answer is $60-13 = \boxed{47}$. □

20. **[30]** Andy and Eddie play a game in which they continuously flip a fair coin. They stop flipping when either they flip tails, heads, and tails consecutively in that order, or they flip three tails in a row. Then, if there has been an odd number of flips, Andy wins, and otherwise Eddie wins. Given that the probability that Andy wins is $\frac{m}{n}$, where m and n are relatively prime positive integers, find $m+n$.
Proposed by Andrew Zhao & Zachary Perry

Solution. $\boxed{39}$

Let H_o be the probability that Andy wins after an "odd heads reset", where there has been a heads flipped on an odd number of flips, and it acts as a reset, so that nothing before that will affect the future probabilities. Let H_e, T_o, T_e denote similar values.

Now, we can create a system of equations for this:

$$H_o = \frac{1}{2}\cdot T_e + \frac{1}{2}\cdot H_e$$
$$H_e = \frac{1}{2}\cdot T_o + \frac{1}{2}\cdot H_o$$
$$T_o = \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 1 + \frac{1}{2}H_o\right) + \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot H_e\right)\right) \implies T_o = \frac{1}{2} + \frac{1}{4}H_o + \frac{1}{8}H_e$$
$$T_e = \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 0 + \frac{1}{2}H_e\right) + \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot\left(\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot H_0\right)\right) \implies T_e = \frac{1}{8} + \frac{1}{4}H_e + \frac{1}{8}H_o$$

Solving this system of equations, we find that $H_o = \frac{11}{25}, T_o = \frac{17}{25}$. Thus, the probability that Andy takes home the champions' belt is $\frac{1}{2}(H_o+T_o) = \frac{14}{25} \implies \boxed{39}$. (Solution by Richard Chen) □

Haikus

A Haiku is a Japanese poem of seventeen syllables, in three lines of five, seven, and five.

21. **[15]** Take five good haikus
Scramble their lines randomly
What are the chances

That you end up with
Five completely good haikus
(With five, seven, five)?

Your answer will be
m over n where m, n
Are numbers such that

m, n positive
Integers where gcd
Of m, n is 1.

Take this answer and
Add the numerator and
Denominator.

Proposed by Jeff Lin

Solution. $\boxed{3004}$

5 lines are seven
Those must be the five middles
We get one over

(choose five from fifteen)
Which is three thousand and three
Add that and the 1. □

22. **[20]** In how many ways
Can you add three integers
Summing seventeen?

Order matters here.
For example, eight, three, six
Is not eight, six, three.

All nonnegative,
Do not need to be distinct.
What is your answer?

Proposed by Derek Gao

Solution. $\boxed{171}$

Stars and bars gives $\binom{17+3-1}{3-1} = \binom{19}{2} = \boxed{171}$. □

23. **[20]** Ada has been told
To write down five haikus plus
Two more every hour.

Such that she needs to

Write down five in the first hour
Seven, nine, so on.

Ada has so far
Forty haikus and writes down
Seven every hour.

At which hour after
She begins will she not have
Enough haikus done?

Proposed by Ada Tsui

Solution. $\boxed{9}$

We want to solve the inequality

$$40+7h<\sum_{i=0}^{h-1}(5+2i) \implies 40+7h<5(h)+2\cdot\frac{(h-1)h}{2} \implies 40<(h-3)h \implies h=\boxed{9}$$

is the earliest hour where this inequality holds. □

24. **[25]** A group of haikus
Some have one syllable less
Sixteen in total.

The group of haikus
Some have one syllable more
Eighteen in total.

What is the largest
Total count of syllables
That the group can't have?

(For instance, a group
Sixteen, seventeen, eighteen
Fifty-one total.)

(Also, you can have
No sixteen, no eighteen
Syllable haikus)

Proposed by Jeff Lin

Solution. $\boxed{127}$

Eighteen equals one
(But modulo seventeen)
Sixteen = minus one

After one-two-eight
Eight times eighteen, or sixteen
gets us ±eight mod 17

Then $\boxed{\text{one-two-seven}}$
We try and do not succeed

To get this number

□

Chandler the Octopus

25. **[15]** Chandler the Octopus is using his eight tentacles to count in binary, where holding up a tentacle signifies a 1 and keeping his tentacle down signifies a 0. Find the sum of the base-10 values of all the numbers in which Chandler is holding up exactly 3 tentacles.

Proposed by Derek Gao

Solution. $\boxed{5355}$

If one tentacle is held up, there are $\binom{7}{2} = 21$ ways to hold up the other two tentacles, so each tentacle is used 21 times. The total value of all of Chandler's tentacles is $2^7 + 2^6 + 2^5 + \cdots + 2^1 + 2^0 = 2^8 - 1 = 255$. Thus, the sum of all values in which Chandler is holding up exactly 3 tentacles is $255 * 21 = \boxed{5355}$. □

26. **[20]** Chandler the Octopus is making a concoction to create the perfect ink. He adds 1.2 grams of melanin, 4.2 grams of enzymes, and 6.6 grams of polysaccharides. But Chandler accidentally added n grams of an extra ingredient to the concoction, Chemical X, to create glue. Given that Chemical X contains none of the three aforementioned ingredients, and the percentages of melanin, enzymes, and polysaccharides in the final concoction are all integers, find the sum of all possible positive integer values of n.

Proposed by Taiki Aiba

Solution. $\boxed{77}$

The problem statement is basically asking for the sum of all n such that $\frac{120}{12+n}$, $\frac{420}{12+n}$, and $\frac{660}{12+n}$ are all integers. This can be found by taking the number of grams of an ingredient, dividing it by the total number of grams in the final concoction, which is $1.2 + 4.2 + 6.6 + n = 12 + n$, and multiplying the result by 100 to account for it being a percentage. Note that a value of n satisfies the conditions if and only if $12 + n$ is a divisor of all three of 120, 420, and 660. Note that $120 = 2^3 \cdot 3 \cdot 5$, $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$, and $660 = 2^2 \cdot 3 \cdot 5 \cdot 11$, so the greatest common divisor of these three numbers is $2^2 \cdot 3 \cdot 5 = 60$. Also, note that every divisor of 60 will also be a divisor of all twhree of 120, 420, and 660. We find that the divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. Of these divisors, only 15, 20, 30, and 60 are greater than 12. Since these divisors are values of $12 + n$, we find that the values of n are 3, 8, 18, and 48, where we subtracted 12 from each of the four divisors. Thus, our final answer is $3 + 8 + 18 + 48 = \boxed{77}$. □

27. **[25]** Chandler the Octopus along with his friends Maisy the Bear and Jeff the Frog are solving LMT problems. It takes Maisy 3 minutes to solve a problem, Chandler 4 minutes to solve a problem and Jeff 5 minutes to solve a problem. They start at 12:00 pm, and Chandler has a dentist appointment from 12:10 pm to 12:30, after which he comes back and continues solving LMT problems. The time it will take for them to finish solving 50 LMT problems, in hours, is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$. **Note: they may collaborate on problems.**

Proposed by Aditya Rao

Solution. $\boxed{102}$

When all 3 work together, they work at a rate of $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$ problems per minute. When Chandler is absent, they work at a rate of $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$ problems per minute. Thus, we have $(x - 20) \cdot \frac{47}{60} + 20 \cdot \frac{8}{15} = 50 \implies x = \frac{118 \cdot 20}{47}$, so the total amount of time we use is $\frac{118 \cdot 20}{47} + 20 = \frac{165 \cdot 20}{47}$ minutes, or $\frac{165 \cdot 20/60}{47} = \frac{55}{47}$ hours. Thus, our answer is $m + n = 55 + 47 = \boxed{102}$. □

Card Games

28. **[15]** Maisy and Jeff are playing a game with a deck of cards with 4 0's, 4 1's, 4 2's, all the way up to 4 9's. You cannot tell apart cards of the same number. After shuffling the deck, Maisy and Jeff each take 4 cards, make the largest 4-digit integer they can, and then compare. The person with the larger 4-digit integer wins. Jeff goes first and draws the cards

$2,0,2,1$ from the deck. Find the number of hands Maisy can draw to beat that, if the order in which she draws the cards matters.

Proposed by Richard Chen

Solution. $\boxed{9897}$

The idea is that any hand which contains a 3 or higher automatically wins, and the only other hand that works is $2,2,1,1$ (since the other 2s were already drawn). There are $10^4 - 3^4$ ways to get a 3 or higher, and 6 ways to get the hand $2,2,1,1$ in some order, but we must remember that there cannot be 3 2s in that first case, which eliminates another 7*4 ways, for a total of $10000 - 81 + 6 - 28 = \boxed{9897}$ □

29. **[20]** Addison and Emerson are playing a card game with three rounds. Addison has the cards 1, 3, and 5, and Emerson has the cards 2, 4, and 6. In advance of the game, both designate each one of their cards to be played for either round one, two, or three. Cards cannot be played for multiple rounds. In each round, both show each other their designated card for that round, and the person with the higher-numbered card wins the round. The person who wins the most rounds wins the game. Let $\frac{m}{n}$ be the probability that Emerson wins, where m and n are relatively prime positive integers. Find $m+n$.

Proposed by Ada Tsui

Solution. $\boxed{11}$

Let Addison designate her cards in the order $1,3,5$.

The total number of ways Emerson can designate her cards is $3! = 6$.

Addison must win two or three of the rounds to win the game.

In the first round, all of Emerson's cards are higher than Addison's card, 1, so Emerson automatically wins this round and Addison must win the remaining two rounds to win the game.

In the second round, only Emerson's card of 2 is lower than Addison's card, 3, so Emerson must play a 2 for Addison to win this round and have a chance to win the game.

In the third round, Emerson's cards of 2 and 4 are lower than Addison's card, 5, but Emerson already used the 2 in the second round, so Emerson must play a 4 for Addison to win this round and the game.

Thus, the only way for Addison to win is for Emerson to designate her cards in the order $6,2,4$. Of course, we are looking for the number of ways for Emerson to win, and that is $6 - 1 = 5$.

The probability Emerson wins is $\frac{5}{6}$, so $m = 5, n = 6$, and $m + n = \boxed{11}$. □

30. **[25]** In a group of 6 people playing the card game Tractor, all 54 cards from 3 decks are dealt evenly to all the players at random. Each deck is dealt individually. Let the probability that no one has at least two of the same card be X. Find the largest integer n such that the nth root of X is rational.

Proposed by Sammy Charney

Solution. $\boxed{}$

Due to the problem having infinitely many solutions, all teams who inputted answers received points. □