Scramble Round Solutions

Lexington High School

April 8, 2017

1. Compute
$$\frac{2+4+\cdots+2018}{1+3+\cdots+2017} + \frac{1+3+\cdots+2017}{2+4+\cdots+2018}.$$

   Proposed by: Yiming Zheng

   ANSWER: $\dfrac{2038181}{1019090}$

   SOLUTION:

2. Find the number of trailing zeros in
$$\prod_{i=1}^{2017} i!.$$

   Proposed by: Yiming Zheng

   ANSWER: 504256

   SOLUTION:

3. Compute the sum of the digits of 20^{17}.

   Proposed by: Nathan Ramesh

   ANSWER: 14

   SOLUTION:

4. Compute $1^2 + 2^2 + \cdots + 2017^2$.

   Proposed by: Peter Rowley

   ANSWER: 2737280785

   SOLUTION:

5. Compute $\lfloor \sqrt{201700} \rfloor$.

   Proposed by: Nathan Ramesh

   ANSWER: 449

   SOLUTION:

6. Compute

$$\sum_{n=1}^{2017} \lfloor \sqrt{n} \rfloor.$$

Proposed by: Nathan Ramesh

ANSWER: $\boxed{59422}$

SOLUTION:

7. Find the last three digits of $2017^{2016} - 2016^{2015} + 2015^{2014} - 2014^{2013}$.

Proposed by: Yiming Zheng

ANSWER: $\boxed{986}$

SOLUTION:

8. Compute

$$\sum_{a,b \in \mathbb{Z}_{\geq 0}} \frac{1}{20^a \cdot 17^b}.$$

Proposed by: Nathan Ramesh

ANSWER: $\boxed{\dfrac{85}{76}}$

SOLUTION:

9. Compute 2017^2.

Proposed by: Nathan Ramesh

ANSWER: $\boxed{4068289}$

SOLUTION:

10. Compute

$$\left(\sum_{n=1}^{20} n \right) \left(\sum_{n=1}^{17} n \right) = (1 + 2 + 3 + \cdots + 19 + 20)(1 + 2 + 3 + \cdots + 16 + 17).$$

Proposed by: Peter Rowley

ANSWER: $\boxed{32130}$

SOLUTION:

11. Compute $1^2 - 2^2 + 3^2 - 4^2 + 5^2 \cdots + 2017^2$.

Proposed by: Peter Rowley

ANSWER: $\boxed{2035153}$

SOLUTION:

12. Compute the number of ordered pairs of nonnegative integers (m, n) which satisfy $20m + 17n \leq 340$.

Proposed by: Nathan Ramesh

ANSWER: $\boxed{190}$

SOLUTION:

13. Compute $20^4 - 17^4$.

    Proposed by: Nathan Ramesh

    ANSWER: 76479

    SOLUTION:

14. Compute $\binom{20}{17}$.

    Proposed by: Nathan Ramesh

    ANSWER: 1140

    SOLUTION:

15. For positive integers n, let $\sigma(n)$ denote the number of positive divisors of n. Find

    $$\max_{1 \le n \le 2017} \sigma(n).$$

    Proposed by: Peter Rowley

    ANSWER: 40

    SOLUTION: $1640 = 2^4 \times 3 \times 5 \times 7$.

16. Compute the number of positive integers less than 2017 which are relatively prime to 2016.

    Proposed by: Nathan Ramesh

    ANSWER: 576

    SOLUTION:

17. Compute $\frac{1}{20} + \frac{1}{17} + \frac{1}{2017}$.

    Proposed by: Nathan Ramesh

    ANSWER: $\dfrac{74969}{685780}$

    SOLUTION:

18. Find the number of primes between 1 and 217, inclusive.

    Proposed by: Nathan Ramesh

    ANSWER: 47

    SOLUTION:

19. Find the last three digits of 17^{2017}.

    Proposed by: Peter Rowley

    ANSWER: 177

    SOLUTION:

20. Compute $20 + 17$.

    Proposed by: Nathan Ramesh

    ANSWER: $\boxed{37}$

    SOLUTION:

21. Compute $20^2 + 19^2 + 18^2 + 17^2$.

    Proposed by: Nathan Ramesh

    ANSWER: $\boxed{1374}$

    SOLUTION:

22. Compute the number of four-digit positive integers with the same digit sum as 2017.

    Proposed by: Peter Rowley

    ANSWER: $\boxed{219}$

    SOLUTION:

23. Find the last three digits of $2017^{2017!} - 2017^{2017}$.

    Proposed by: Yiming Zheng

    ANSWER: $\boxed{824}$

    SOLUTION:

24. Compute
$$\sum_{n=1}^{2017} n = 1 + 2 + 3 + \cdots + 2016 + 2017.$$

    Proposed by: Nathan Ramesh

    ANSWER: $\boxed{2035153}$

    SOLUTION:

25. Compute
$$\sum_{a,b \in \mathbb{Z}_{\geq 0}} \binom{20}{a}\binom{17}{b}.$$

    Proposed by: Nathan Ramesh

    ANSWER: $\boxed{137438953472}$

    SOLUTION:

26. Compute $20^2 - 17^2$.

    Proposed by: Nathan Ramesh

    ANSWER: $\boxed{111}$

    SOLUTION:

27. Compute
$$\sum_{n=1}^{2017} n \cdot (-1)^n = -1 + 2 - 3 + 4 \cdots + 2016 - 2017.$$

Proposed by: Nathan Ramesh

ANSWER: $\boxed{-1009}$

SOLUTION:

28. When all of the numbers from 1 to 2017 are written out, how many times does the digit 1 appear?

Proposed by: Peter Rowley

ANSWER: $\boxed{1610}$

SOLUTION:

29. Find the sum of all four-digit positive integers that share at least one common digit with 2017.

Proposed by: Yiming Zheng

ANSWER: $\boxed{49456615}$

SOLUTION:

30. Compute the number of positive integers less than 2017 which are relatively prime to 2017.

Proposed by: Peter Rowley

ANSWER: $\boxed{2016}$

SOLUTION: