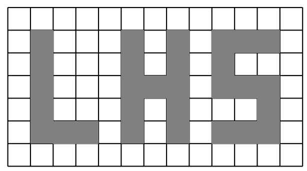
Speed Round

LMT "Fall"

December 17, 2022

1. **[6]** Each box represents 1 square unit. Find the area of the shaded region.



2. **[6]** Evaluate $(3^3)\sqrt{5^2-2^4}-5\cdot 9$.

3. **[6]** Find the last two digits of 21^3.

4. **[6]** Let L, M, and T be distinct prime numbers. Find the least possible odd value of $L+M+T$.

5. **[6]** Two circles have areas that sum to 20π and diameters that sum to 12. Find the radius of the smaller circle.

6. **[6]** Zach and Evin each independently choose a date in the year 2022, uniformly and randomly. The probability that at least one of the chosen dates is December 17, 2022 can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find A.

7. **[6]** Let L be a list of 2023 real numbers with median m. When any two numbers are removed from L, its median is still m. Find the greatest possible number of distinct values in L.

8. **[6]** Some children and adults are eating a delicious pile of sand. Children comprise 20% of the group and combined, they consume 80% of the sand. Given that on average, each child consumes N pounds of sand and on average, each adult consumes M pounds of sand, find $\frac{N}{M}$.

9. **[6]** An integer N is chosen uniformly and randomly from the set of positive integers less than 100. The expected number of digits in the base-10-representation of N can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A+B$.

10. **[6]** Dunan is taking a calculus course in which the final exam counts for 15% of the total grade. Dunan wishes to have an A in the course, which is defined as a grade of 93% or above. When counting everything but the final exam, he currently has a 92% in the course. What is the minimum integer grade Dunan must get on the final exam in order to get an A in the course?

11. **[6]** Norbert, Eorbert, Sorbert, and Worbert start at the origin of the Cartesian Plane and walk in the positive y, positive x, negative y, and negative x directions respectively at speeds of 1, 2, 3, and 4 units per second respectively. After how many seconds will the quadrilateral with a vertex at each person's location have area 300?

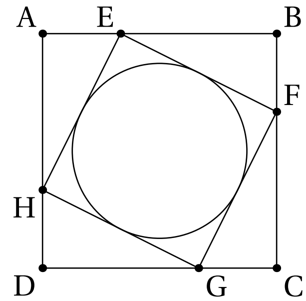
12. **[6]** Find the sum of the unique prime factors of 1020201.

13. **[6]** HacoobaMatata rewrites the base-10 integers from 0 to 30 inclusive in base 3. How many times does he write the digit 1?

14. **[6]** The fractional part of x is $\frac{1}{7}$. The greatest possible fractional part of x^2 can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A+B$.

15. **[6]** For how many integers x is $-2x^2+8 \geq x^2-3x+2$?

16. **[6]** In the figure below, circle ω is inscribed in square $EFGH$, which is inscribed in unit square $ABCD$ such that $\overline{EB} = 2\overline{AE}$. If the minimum distance from a point on ω to $ABCD$ can be written as $\frac{P-\sqrt{Q}}{R}$ with Q square-free, find $10000P + 100Q + R$.



17. **[6]** There are two base number systems in use in the LHS Math Team. One member writes "13 people use my base, while 23 people use the other, base 12." Another member writes "out of the 34 people in the club, 10 use both bases while 9 use neither." Find the sum of all possible numbers of Math Team members, as a regular decimal number.

18. **[6]** Sam is taking a test with 100 problems. On this test the questions gradually get harder in such a way that for question i, Sam has a $\frac{(101-i)^2}{100}$% chance to get the question correct. Suppose the expected number of questions Sam gets correct can be written as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.

19. **[6]** In an ordered 25-tuple, each component is an integer chosen uniformly and randomly from $\{1,2,3,4,5\}$. Ephram and Zach both copy this tuple into a 5×5 grid, both starting from the top-left corner. Ephram writes five components from left to right to fill one row before continuing down to the next row. Zach writes five components from top to bottom to fill one column before continuing right to the next column. Find the expected number of spaces on their grids where Zach and Ephram have the same integer written.

20. **[6]** In $\triangle ABC$ with circumcenter O and circumradius 8, $BC = 10$. Let r be the radius of the circle that passes through O and is tangent to BC at C. The value of r^2 can be written as $\frac{m}{n}$ for relatively prime positive integers m and n. Find $1000m + n$.

21. **[6]** Find the number of integer values of n between 1 and 100 inclusive such that the sum of the positive divisors of $2n$ is at least 220% of the sum of the divisors of n.

22. **[6]** Twenty urns containing one ball each are arranged in a circle. Ernie then moves each ball either 1, 2 or 3 urns clockwise, chosen independently, uniformly, and randomly. The expected number of empty urns after this process is complete can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.

23. **[6]** Hannah the cat begins at 0 on a number line. Every second, Hannah jumps 1 unit in the positive or negative direction, chosen uniformly at random. After 7 seconds, Hannah's expected distance from 0, in units, can be expressed as $\frac{A}{B}$ for relatively prime positive integers A and B. Find $1000A + B$.

24. **[6]** Find the product of all primes $p < 30$ for which there exists an integer n such that p divides $n + (n+1)^{-1} \pmod{p}$.

25. **[6]** In quadrilateral $ABCD$, $\angle ABD = \angle CBD = \angle CAD$, $AB = 9$, $BC = 6$, and $AC = 10$. The area of $ABCD$ can be expressed as $\frac{P\sqrt{Q}}{R}$ with Q squarefree and P and R relatively prime. Find $10000P + 100Q + R$.