3rd Annual Lexington Mathematical Tournament
Individual Round

Saturday, May 5, 2012

1. Evaluate $1! + 2! + 3! + 4! + 5!$ (where $n!$ is the product of all integers from 1 to n, inclusive).

2. Harold opens a pack of Bertie Bott's Every Flavor Beans that contains 10 blueberry, 10 watermelon, 3 spinach and 2 earwax-flavored jelly beans. If he picks a jelly bean at random, then what is the probability that it is not spinach-flavored?

3. Find the sum of the positive factors of 32 (including 32 itself).

4. Carol stands at a flag pole that is 21 feet tall. She begins to walk in the direction of the flag's shadow to say hi to her friends. When she has walked 10 feet, her shadow passes the flag's shadow. Given that Carol is exactly 5 feet tall, how long in feet is her shadow?

5. A solid metal sphere of radius 7 cm is melted and reshaped into four solid metal spheres with radii 1, 5, 6, and x cm. What is the value of x?

6. Let $A = (2, -2)$ and $B = (-3, 3)$. If $(a, 0)$ and $(0, b)$ are both equidistant from A and B, then what is the value of $a + b$?

7. For every flip, there is an x^2 percent chance of flipping heads, where x is the number of flips that have already been made. What is the probability that my first three flips will all come up tails?

8. Consider the sequence of letters Z W Y X V. There are two ways to modify the sequence: we can either swap two adjacent letters or reverse the entire sequence. What is the least number of these changes we need to make in order to put the letters in alphabetical order?

9. A square and a rectangle overlap each other such that the area inside the square but outside the rectangle is equal to the area inside the rectangle but outside the square. If the area of the rectangle is 169, then find the side length of the square.

10. If $A = 50\sqrt{3}$, $B = 60\sqrt{2}$, and $C = 85$, then order A, B, and C from least to greatest.

11. How many ways are there to arrange the letters of the word RACECAR? (Identical letters are assumed to be indistinguishable.)

12. A cube and a regular tetrahedron (which has four faces composed of equilateral triangles) have the same surface area. Let r be the ratio of the edge length of the cube to the edge length of the tetrahedron. Find r^2.

13. Given that $x^2 + x + \frac{1}{x} + \frac{1}{x^2} = 10$, find all possible values of $x + \frac{1}{x}$.

14. Astronaut Bob has a rope one unit long. He must attach one end to his spacesuit and one end to his stationary spacecraft, which assumes the shape of a box with dimensions $3 \times 2 \times 2$. If he can attach and re-attach the rope onto any point on the surface of his spacecraft, then what is the total volume of space outside of the spacecraft that Bob can reach? Assume that Bob's size is negligible.

15. Triangle ABC has $AB = 4$, $BC = 3$, and $AC = 5$. Point B is reflected across $\overline{AC}$ to point B'. The lines that contain AB' and BC are then drawn to intersect at point D. Find AD.

16. Consider a rectangle $ABCD$ with side lengths 5 and 12. If a circle tangent to all sides of $\triangle ABD$ and a circle tangent to all sides of $\triangle BCD$ are drawn, then how far apart are the centers of the circles?

17. An increasing geometric sequence $a_0, a_1, a_2, \ldots$ has a positive common ratio. Also, the value of $a_3 + a_2 - a_1 - a_0$ is equal to half the value of $a_4 - a_0$. What is the value of the common ratio?

18. In triangle ABC, $AB = 9$, $BC = 11$, and $AC = 16$. Points E and F are on $\overline{AB}$ and $\overline{BC}$, respectively, such that $BE = BF = 4$. What is the area of triangle CEF?

19. Xavier, Yuna, and Zach are running around a circular track. The three start at one point and run clockwise, each at a constant speed. After 8 minutes, Zach passes Xavier for the first time. Xavier first passes Yuna for the first time in 12 minutes. After how many *seconds* since the three began running did Zach first pass Yuna?

20. How many unit fractions are there such that their decimal equivalent has a cycle of 6 repeating integers? Exclude fractions that repeat in cycles of 1, 2, or 3.