

2nd Annual Lexington Mathematical Tournament - Guts Round Part 1

April 2, 2011

1. [5] Compute $(1 - 2(3 - 4(5 - 6)))(7 - (8 - 9))$.
2. [5] How many numbers are in the set $\{20, 21, 22, \dots, 88, 89\}$?
3. [5] Three times the complement of the supplement of an angle is equal to 60 degrees less than the angle itself. Find the measure of the angle in degrees.

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4. [5] A positive number is decreased by 10%, then decreased by 20%, and finally increased by 30%. By what percent has this number changed from the original? Give a positive answer for a percent increase and a negative answer for a percent decrease.
5. [5] What is the area of the triangle with vertices at $(2, 3)$, $(8, 11)$, and $(13, 3)$?
6. [5] There are three bins, each containing red, green, and/or blue pens. The first bin has 0 red, 0 green, and 3 blue pens, the second bin has 0 red, 2 green, and 4 blue pens, and the final bin has 1 red, 5 green, and 6 blue pens. What is the probability that if one pen is drawn from each bin at random, one of each color pen will be drawn?

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7. [6] If a and b are positive integers and $a^2 - b^2 = 23$, what is the value of a ?
8. [6] Find the prime factorization of the greatest common divisor of $2^3 \cdot 3^2 \cdot 5^5 \cdot 7^4$ and $2^4 \cdot 3^1 \cdot 5^2 \cdot 7^6$.
9. [6] Given that

$$\begin{aligned}a + 2b + 3c &= 5 \\2a + 3b + c &= -2 \\3a + b + 2c &= 3,\end{aligned}$$

find $3a + 3b + 3c$.

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10. [6] How many positive integer divisors does 11^{20} have?
11. [6] Let α be the answer to problem 10. Find the real value of x such that $2^{x-5} = 64^{x/\alpha}$.
12. [6] Let β be the answer to problem 11. Triangle LMT has a right angle at M , $LM = \beta$, and $LT = 4\beta - 3$. If Z is the midpoint of LT , what is the length MZ ?

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13. [7] Simplify $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21}$.
14. [7] Given that $x + y = 7$ and $x^2 + y^2 = 29$, what is the sum of the reciprocals of x and y ?
15. [7] Consider a rectangle $ABCD$ with side lengths $AB = 3$ and $BC = 4$. If circles are inscribed in triangles ABC and BCD , how far are the centers of the circles from each other?

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16. [7] Evaluate

$$\frac{2!}{1!} + \frac{3!}{2!} + \frac{4!}{3!} + \cdots + \frac{99!}{98!} + \frac{100!}{99!}.$$

17. [7] Let $ABCD$ be a square of side length 2. A semicircle is drawn with diameter \overline{AC} that passes through point B . Find the area of the region inside the semicircle but outside the square.
18. [7] For how many positive integer values of k is $\frac{37k - 30}{k}$ a positive integer?

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19. [8] Two parallel planar slices across a sphere of radius 25 create cross sections of area 576π and 225π . What is the maximum possible distance between the two slices?
20. [8] How many positive integers cannot be expressed in the form $3l + 4m + 5t$, where l , m , and t are nonnegative integers?
21. [8] In April, a fool is someone who is fooled by a classmate. In a class of 30 students, 14 people were fooled by someone else and 29 people fooled someone else. What is the largest positive integer n for which we can guarantee that at least one person was fooled by at least n other people?

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22. [8] Let

$$S = 4 + \frac{12}{4 + \frac{12}{4 + \frac{12}{4 + \dots}}}$$

Evaluate $4 + \frac{12}{S}$.

23. [8] Jonathan is buying bananagram sets for \$11 each and flip-flops for \$17 each. If he spends \$227 on purchases for bananagram sets and flip-flops, what is the total number of bananagram sets and flip-flops he bought?
24. [8] Alan has a 3×3 array of squares. He starts removing the squares one at a time such that each time he removes one square, all remaining squares share a side with at least two other remaining squares. What is the maximum number of squares Alan can remove?

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25. [9] Let \mathcal{S} be the region bounded by the lines $y = x/2$, $y = -x/2$, and $x = 6$. Pick a random point $P = (x, y)$ in \mathcal{S} and translate it 3 units right to $P' = (x + 3, y)$. What is the probability that P' is in \mathcal{S} ?
26. [9] A triangle with side lengths 17, 25, and 28 has a circle centered at each of its three vertices such that the three circles are mutually externally tangent to each other. What is the combined area of the circles?
27. [9] Find all ordered pairs (x, y) of integers such that $x^2 - 2x + y^2 - 6y = -9$.

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28. [11] In how many ways can the letters in the word SCHAFKOPF be arranged if the two F's cannot be next to each other and the A and the O must be next to each other?
29. [11] Let a sequence a_0, a_1, a_2, \dots be defined by $a_0 = 20$, $a_1 = 11$, $a_2 = 0$, and for all integers $n \geq 3$,

$$a_n + a_{n-1} = a_{n-2} + a_{n-3}.$$

Find the sum $a_0 + a_1 + a_2 + \dots + a_{2010} + a_{2011}$.

30. [11] Find the sum of all positive integers b such that the base b number 190_b is a perfect square.

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31. [13] Find all real values of x such that $\sqrt[3]{4x-1} + \sqrt[3]{4x+1} = \sqrt[3]{8x}$.
32. [13] Right triangle ABC has a right angle at B . The angle bisector of $\angle ABC$ is drawn and extended to a point E such that $\angle ECA = \angle ACB$. Let F be the foot of the perpendicular from E to ray \overrightarrow{BC} . Given that $AB = 4$, $BC = 2$, and $EF = 8$, find the area of triangle ACE .
33. [13] You are the soul in the southwest corner of a four by four grid of distinct souls in the Fields of Asphodel. You move one square east and at the same time all the other souls move one square north, south, east, or west so that each square is now reoccupied and no two souls switched places directly. How many end results are possible from this move?

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34. [≤ 15] A *Pythagorean triple* is an ordered triple of positive integers (a, b, c) with $a < b < c$ and $a^2 + b^2 = c^2$. A *primitive Pythagorean triple* is a Pythagorean triple where all three numbers are relatively prime to each other. Find the number of primitive Pythagorean triples in which all three members are less than 100,000. If P is the true answer and A is your team's answer to this problem, your score will be $\max\left\{15 - \frac{|A - P|}{500}, 0\right\}$, rounded to the nearest integer.
35. [≤ 15] According to the Enable2k North American word list, how many words in the English language contain the letters L, M, T in order but not necessarily together? If A is your team's answer to this problem and W is the true answer, the score you will receive is $\max\left\{15 - 100\left|\frac{A}{W} - 1\right|, 0\right\}$, rounded to the nearest integer.
36. [≤ 15] Write down 5 positive integers less than or equal to 42. For each of the numbers written, if no other teams put down that number, your team gets 3 points. Otherwise, you get 0 points. Any number written that does not satisfy the given requirement automatically gets 0 points.