

1st Annual Lexington Mathematical Tournament Theme Round

April 3, 2010

1 Cheetahs

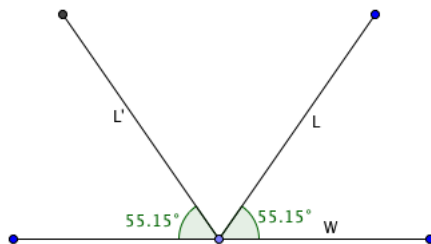
Cheetahs in my dresser,
Cheetahs in my hair.
Cheetahs in my pants,
Cheetahs everywhere!

-A poem by J. Samuel Trabucco, Esq.

1. J has several cheetahs in his dresser, which has 7 drawers, such that each drawer has the same number of cheetahs. He notices that he can take out one drawer, and redistribute all of the cheetahs (including those in the removed drawer) in the remaining 6 drawers such that each drawer still has an equal number of cheetahs as the other drawers. If he has at least one cheetah, what is the smallest number of cheetahs that he can have?
2. J has 53 cheetahs in his hair, which he will put in 10 cages. Let A be the number of cheetahs in the cage with the largest number of cheetahs (there could be a tie, but in this case take the number of cheetahs in one of the cages involved in the tie). Find the least possible value of A .
3. J has 98 cheetahs in his pants, some of which are male and the rest of which are female. He realizes that three times the number of male cheetahs in his pants is equal to nine more than twice the number of female cheetahs. How many male cheetahs are in his pants?
4. Because J's cheetahs are everywhere, they are now running away. A particularly unintelligent one starts to run in a 720 mile loop at 80 miles per hour. J immediately starts to chase after it, starting at the same point, at 10 miles per hour at 12:00 PM, but realizes one hour later that it would be more wise to turn around and run in the opposite direction in the loop, so he does this. Assuming both maintain a constant speed, at what time do J and the cheetah collide? Round to the nearest minute, and be sure to include AM or PM.
5. Once J and his cheetah collide, J dies a very slow and painful death. The cheetahs come back for his funeral, which is held in a circular stadium with 10 rows. The first row has 10 seats in a circle, and each subsequent row has 3 more seats. However, no two adjacent seats may be occupied due to the size of the cheetahs. What is the maximum number of cheetahs that can fit in the stadium?

2 Lasers

Laser beams are known for reflecting off solid objects. Whenever a laser beam hits a straight, solid wall, it reflects off in the opposite direction, at an angle to the wall that is equal to the angle at which it hits, as shown at the top of the next page.



The path of the laser L meets the wall, W , at an angle of 55.15 degrees coming from the right. Then, L reflects off W to a redirected path, L' , at an angle of 55.15 degrees going off to the left. In particular, if L were to meet W at an angle of 90 degrees, then L' would follow L , going backwards (straight up).

6. Given a square $ABCD$, with $AB = 1$, mark the midpoints M and N of AB and BC , respectively. A laser beam shot from M to N , and the beam reflects off BC , CD , DA , and comes back to M . This path encloses a smaller area inside square $ABCD$. Find this area.

7. Given a rectangle $EFGH$, with $EF = 3$ and $FG = 40$, mark a point P on FG such that $FP = 4$. A laser beam is shot from E to P , which then reflects off FG , then EH , then FG , etc. Once it reaches some point on GH , the beam is absorbed; it stops reflecting. How far does the beam travel?

8. Given a rectangle $EFGH$ with $EF = 3$ and $FG = 2010$, mark a point P on FG such that $FP = 4$. A laser beam is shot from E to P , which then reflects off FG , then EH , then FG , etc. Once it reaches some point on GH , the beam is absorbed; it stops reflecting. How far does the beam travel?

9. Given a triangle XYZ with $\angle Y = 90^\circ$, $XY = 1$, and $XZ = 2$, mark a point Q on YZ such that $\frac{ZQ}{ZY} = \frac{1}{3}$. A laser beam is shot from Q perpendicular to YZ , and it reflects off the sides of XYZ indefinitely. How many bounces does it take for the laser beam to get back to Q for the first time (not including the release from Q and the return to Q)?

10. Given a triangle XYZ with $\angle Y = 90^\circ$, $XY = 1$, and $XZ = 2$, mark a point Q on YZ such that $\frac{ZQ}{ZY} = \frac{1}{3}$. A laser beam is shot from Q perpendicular to YZ , and it reflects off the sides of XYZ indefinitely. How far has the laser traveled when it reaches its 2010th bounce?

3 Rock Paper Scissors

In the game of Rock Paper Scissors, typically two opponents make, with their hand, either a rock, a piece of paper, or a pair of scissors. If the two players play rock and scissors, the one who plays rock wins. If they play scissors and paper, the one who plays scissors wins, and if they play paper and rock, the one who plays paper wins. If they play the same thing, the result is a tie.

11. Al and Bob are playing Rock Paper Scissors. Al plays rock. What is the probability that Al wins, given that Bob plays randomly and has an equal probability of playing rock, paper, and scissors?

12. In a given game, what is the probability of a tie, given that both players play randomly and with an equal probability of playing rock, paper, and scissors?

13. Al and Bob play Rock Paper Scissors until someone wins a game. What is the probability that this happens on the sixth game?

14. Al and Bob are joined by Carl and D'Angelo, and they decide to play a team game of Rock Paper Scissors. A game is called *perfect* if some two of the four play the same thing, and the other two also play the same thing, but something different. For example, an example of a perfect game would be Al and Bob playing rock, and Carl and D'Angelo playing scissors, but if all four play paper, we do not have a perfect game. What is the probability of a perfect game?

15. Al is bored of Rock Paper Scissors, and wants to invent a new game: Z-Y-X-W-V. Two players, each choose to play either Z, Y, X, W, or V. If they play the same thing, the result is a tie. However, Al must come up with a 'pecking order', that is, he must decide which plays beat which. For each of the 10 pairs of distinct plays that the two players can make, Al randomly decides a winner. For example, he could decide that W beats Y and that Z beats X, etc. What is the probability that after Al makes all of these 10 choices, the game is balanced, that is, playing each letter results in an equal probability of winning?