## 1st Annual Lexington Mathematical Tournament - Guts Round Part 1

April 3, 2010

1. **[5]** Compute $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.

2. **[5]** If you increase a number $X$ by 20%, you get $Y$. By what percent must you decrease $Y$ to get $X$?

3. **[5]** A circle has circumference $8\pi$. Determine its radius.

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4. **[5]** The perimeter of a square is equal in value to its area. Determine the length of one of its sides.

5. **[5]** Big Welk writes the letters of the alphabet in order, and starts again at A each time he gets to Z. What is the $4^3$-rd letter that he writes down?

6. **[5]** Al travels for 20 miles per hour rolling down a hill in his chair for two hours, then four miles per hour climbing a hill for six hours. What is his average speed, in miles per hour?

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7. **[6]** A team of four students goes to LMT, and each student brings a lunch. However, on the bus, the students' lunches get mixed up, and during lunch time, each student chooses a random lunch to eat (no two students may eat the same lunch). What is the probability that each student chooses his or her own lunch correctly?

8. **[6]** The integer 111111 is the product of five prime numbers. Determine the sum of these prime numbers.

9. **[6]** A trapezoid has bases with lengths equal to 5 and 15 and legs with lengths equal to 13 and 13. Determine the area of the trapezoid.

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10. **[6]** A two digit prime number is such that the sum of its digits is 13. Determine the integer.

11. **[6]** Carl, James, Saif, and Ted play several games of two-player For The Win on the Art of Problem Solving website. If, among these games, Carl wins 5 and loses 0, James wins 4 and loses 2, Saif wins 1 and loses 6, and Ted wins 4, how many games does Ted lose?

12. **[6]** $a, b, c, d, e$ are equal to $1, 2, 3, 4, 5$ in some order, such that no two of $a, b, c, d, e$ are equal to the same integer. Given that $b \leq d$, $c \geq a$, $a \leq e$, $b \geq e$, and that $d \neq 5$, determine the value of $a^b + c^d + e$.

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13. **[7]** A circle with center $O$ has radius 5, and has two points $A, B$ on the circle such that $\angle AOB = 90°$. Rays $OA$ and $OB$ are extended to points $C$ and $D$, respectively, such that $AB$ is parallel to $CD$, and the length $CD$ is 200% more than the radius of circle $O$.. Determine the length $AC$.

14. **[7]** Seongcheol has 3 red shirts and 2 green shirts, such that he cannot tell the difference between his three red shirts and he similarly cannot tell the difference between his two green shirts. In how many ways can he hang them in a row in his closet, given that he does not want the two green shirts next to each other?

15. **[7]** Determine the number of ordered pairs $(x, y)$ with $x$ and $y$ integers between -5 and 5, inclusive, such that $(x + y)(x + 3y) = (x + 2y)^2$.

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16. **[7]** Al has three bags, each with three marbles each. Bag 1 has two blue marbles and one red marble, Bag 2 has one blue marble and two red marbles, and Bag 3 has three red marbles. He chooses two distinct bags at random, then one marble at random from each of the chosen bags. What is the probability that he chooses two blue marbles?

17. **[7]** Determine the sum of the two largest prime factors of the integer $89! + 90!$.

18. **[7]** Congruent unit circles intersect in such a way that the center of each circle lies on the circumference of the other. Let $R$ be the region in which the two circles overlap. Determine the perimeter of $R$.

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19. **[8]** Let $f(x) = x^2 - 2x + 1$. For some constant $k$, $f(x + k) = x^2 + 2x + 1$ for all real numbers $x$. Determine the value of $k$.

20. **[8]** Three vertices of a parallelogram are $(2, -4)$, $(-2, 8)$, and $(12, 7)$. Determine the sum of the three possible $x$-coordinates of the fourth vertex.

21. **[8]** Jae and Yoon are playing SunCraft. The probability that Jae wins the $n$-th game is $\frac{1}{n+2}$. What is the probability that Yoon wins the first six games, assuming there are no ties?

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22. **[8]** Two circles, $\omega_1$ and $\omega_2$, intersect at $X$ and $Y$. The segment between their centers intersects $\omega_1$ and $\omega_2$ at $A$ and $B$, respectively, such that $AB = 2$. Given that the radii of $\omega_1$ and $\omega_2$ are 3 and 4, respectively, find $XY$.

23. **[8]** In how many ways can six marbles be placed in the squares of a 6-by-6 grid such that no two marbles lie in the same row or column?

24. **[8]** Let $ABC$ be an equilateral triangle with $AB = 1$. Let $M$ be the midpoint of $BC$, and let $P$ be on segment $AM$ such that $AM/MP = 4$. Find $BP$.

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*The answers in this section all depend on each other.*

25. **[10]** Let $C$ be the answer to Problem 27. What is the $C$-th smallest positive integer with exactly four positive factors?

26. **[10]** Let $A$ be the answer to Problem 25. Determine the absolute value of the difference between the two positive integer roots of the quadratic equation $x^2 - Ax + 48 = 0$.

27. **[10]** Let $B$ be the answer to Problem 26. Compute the smallest integer greater than $\frac{B}{\pi}$.

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28. **[10]** Two knights placed on distinct square of an $8 \times 8$ chessboard, whose squares are unit squares, are said to *attack each other* if the distance between the centers of the squares on which the knights lie is $\sqrt{5}$. In how many ways can two identical knights be placed on distinct squares of an $8 \times 8$ chessboard such that they do NOT attack each other?

29. **[10]** Let $S$ be the set of integers that represent the number of intersections of some four distinct lines in the plane. List the elements of $S$ in ascending order.

30. **[10]** Rick has 7 books on his shelf: three identical red books, two identical blue books, a yellow book, and a green book. Dave accidentally knocks over the shelf and has to put the books back on in the same order. He knows that none of the red books were next to each other and that the yellow book was one of the first four books on the shelf, counting from the left. If Dave puts back the books according to the rules, but otherwise randomly, what is the probability that he puts the books back correctly?

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31. **[12]** In how many ways can each of the integers 1 through 11 be assigned one of the letters L, M, and T such that consecutive multiples of $n$, for any positive integer $n$, are not assigned the same letter?

32. **[12]** Compute the infinite sum $\frac{1^3}{2^1} + \frac{2^3}{2^2} + \frac{3^3}{2^3} + \cdots + \frac{n^3}{2^n} + \cdots$.

33. **[12]** Let $ABCD$ be a unit square. $E$ and $F$ trisect $AB$ such that $AE < AF$. $G$ and $H$ trisect $BC$ such that $BG < BH$. $I$ and $J$ bisect $CD$ and $DA$, respectively. Let $HJ$ and $EI$ meet at $K$, and let $GJ$ and $FI$ meet at $L$. Compute the length $KL$.

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34. **[16]** A *prime power* is an integer of the form $p^k$, where $p$ is a prime and $k$ is a non-negative integer. How many prime powers are there less than or equal to $10^6$? Your score will be $16 - 80\left|\frac{\text{Your Answer}}{\text{Actual Answer}} - 1\right|$ rounded to the nearest integer or 0, whichever is higher.

35. **[16]** Consider a set of 6 fixed points in the plane, with no three collinear. Between some pairs of these points, we may draw one arrow from one point to the other. How many possible configurations of arrows are there such that if there is an arrow from point $A$ to point $B$ and an arrow from $B$ to $C$, then there is an arrow from $A$ to $C$? Your score will be $16 - \frac{1}{800}|\text{Your Answer} - \text{Actual Answer}|$ rounded to the nearest integer or zero, whichever is higher.

36. **[16]** Write down one of the following integers: 1, 2, 4, 8, 16. If your team is the only one that submits this integer, you will receive that number of points; otherwise, you receive zero.